



# Benchmark problem: an air brake model for trains

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## Abstract

This paper proposes a simplified hybrid model of a freight train equipped with an air brake. The control of such a system and the enforcement of numerous safety constraints constitute a relevant benchmark to evaluate tools for proving safety requirements in hybrid systems.

**Category:** industrial **Difficulty:** high

## 1 Introduction

In a wide range of applications, ubiquitous automatic controllers have been replacing human operators. Automatic train operation (ATO), while not so well known as self-driving cars, has also been widely studied and implemented in a few cases, such as in subways and monorails [8]. One main problem in ATO relies on the control of tractive and braking effort, where a desired speed profile is to be tracked while satisfying constraints on the actuation and train acceleration. Train braking in particular is a fairly complex process which takes place in a distributed way at different points of the train cars.

Trains are typically equipped with different braking systems, depending on the characteristics of the trains, loads, operating speed and expected traffic conditions, among others. In this paper we focus on air brakes for two reasons: first, they represent a key actuator to ensure the safety operation of a train. In fact, several serious train accidents can be tracked back to failures in the air brake system (i.e., [7]). Second, describing its behaviour in an accurate way requires a hybrid model, which poses a challenging problem from a control and verification perspective. Air brakes date back to 1868 [5], and are mostly based on compressed air. Air brakes resort to air pressure variations to command a change in the braking force. These brakes can be used as service brakes, during regular operation, and also as emergency brakes.

Many modelling efforts are available in the literature (e.g., [9], [1]). In this report we propose a simplified model of a train, of an air brake, and of the engine moving the train. While the proposed models are fairly simplified, we believe they reflect the complexity and hybrid nature of train systems and fit the purpose of the workshop. To the best of our knowledge, existing ATO techniques do not approach the problem in a systematic way and overlook the hybrid behaviour of the air brake.

The remainder of the paper is structured as follows: Section 2 describes the dynamics of the train and the air brake. Section 3 defines the control specification as an optimisation problem

under safety requirements. Section 4 describes the provided code. Finally Section 5 proposes several ways to increase the complexity the problem, both from a modelling perspective (via model extensions to include more realistic aspects) and from a specification point of view.

## 2 A simplified model of a train with hydraulic brake

We consider a simplified model of a freight train and of an air brake. The goal is to obtain a conceptually simple system that exhibits its hybrid nature and where key control issues are highlighted. In the proposed models, we neglect (among others) the effects of a distributed mass and pull-traction forces that the wagons generate between each other, the effects that gravity has on different parts of the train, and the distributed nature of the braking force. Likewise, we ignore other braking subsystems that might be present in trains. Possible extensions to make the simplified model more realistic are discussed in Section 5. The overall model is composed of the nonlinear dynamics of the train, the dynamics of the engine, and the hybrid dynamics of the brake.

### 2.1 Train model

We model the train as a point mass and consider the case where the train moves along a 1-dimensional manifold. With  $\xi(t)$  we denote the position of the train at time  $t$  along the path, and with  $v(t)$  we denote the speed of the train at time  $t$ . The power generated by the engine is denoted by  $p(t)$ . The speed of the train is affected by the power of the engine, the effects of air and of rolling resistance, the force generated by the air brake, and the effect of gravity. The effects of air and of rolling resistance is typically called *propulsion resistance* and, as discussed in [6], its modulus can be modelled as a second order polynomial of the absolute value of the speed of the train. The modulus of the propulsion resistance is denoted with  $\mu(|v(t)|)$ , where  $|x|$  represents the absolute value of  $x$ . With  $b(t)$  we denote the magnitude of the force generated by the air brake. The effect of gravity is modelled via the term  $-g \sin(\theta(\xi))$ , where  $g$  is the gravitational acceleration and  $\theta(\xi)$  is the angle of the path at point  $\xi$  with respect to the horizontal line. The dynamics of the train can now be written as

$$\dot{\xi} = v \tag{1}$$

$$\dot{v} = \frac{1}{M} \left( f(p, v) - \text{sgn}(v) \left( \mu(|v|) + b \right) - g \sin(\theta(\xi)) \right), \tag{2}$$

where  $M$  represents the mass of the train,  $f(p, v)$  is the force generated by the engine, which is a function of  $p(t)$  and  $v(t)$  as defined below, and  $\text{sgn}(\cdot)$  is the sign function.<sup>1</sup> For ease of notation, time dependency is neglected in (1) and (2).

The force generated by the engine is inverse proportional to the speed of the train only for speed values higher than a threshold  $\bar{V}$ . For speed values lower than  $\bar{V}$ , the maximum force generated by the engine stays constant for constant power. A possible model for the force generated by the engine can be written as

$$f(t) = \begin{cases} \frac{p(t)}{\bar{V}} & \text{if } |v(t)| < \bar{V} \\ \frac{p(t)}{|v(t)|} & \text{otherwise} \end{cases} \tag{3}$$

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<sup>1</sup>Let  $x$  be a real number, we define  $\text{sgn}(x)$  equal to  $-1$  if  $x < 0$ ,  $0$  if  $x = 0$ , and  $1$  if  $x > 0$ .

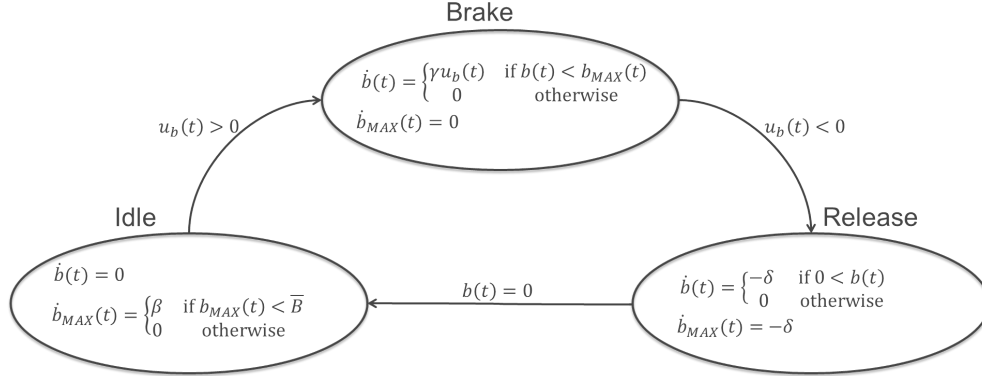


Figure 1: Transitions among discrete states of the brake.

The power generated by the engine is bounded above by  $\bar{P}$  and below by  $\underline{P}$ . In general,  $\underline{P}$  takes negative values since engines can be used as dynamic brakes. In the proposed simplified model, the power of the engine follows a dynamic equation which can be written as:

$$\dot{p}(t) = \begin{cases} \alpha u_d(t) & \text{if } \underline{P} < p(t) < \bar{P} \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

where  $u_d(t)$  is a control input bounded in the interval  $[-1, 1]$  and  $\alpha > 0$  is a constant that depends on the characteristics of the engine.

## 2.2 Brake model

The dynamics of the force generated by the brake is modelled as a hybrid system. The set of discrete states of the brake is composed of three elements named *Idle*, *Brake*, and *Release* which are associated with the indexes 0, 1, and 2, respectively. The discrete state is denoted with the variable  $\zeta(t)$ . The continuous states of the brake model are the force of the brake  $b(t)$  and the maximum force that the brake can generate, denoted with  $b_{MAX}(t)$ . The control input of the brake is denoted with the variable  $u_b(t)$  which is bounded in the interval  $[-1, 1]$ .

When in *Idle*, the brake generates no force and it recharges itself. The maximum force that the brake can generate increases over time up to the physical limit denoted with  $\bar{B}$ . We can then write:

$$\begin{aligned} \dot{b}(t) &= 0 \\ \dot{b}_{MAX}(t) &= \begin{cases} \beta & \text{if } b_{MAX}(t) < \bar{B} \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \quad (5)$$

with  $\beta > 0$  being determined by the characteristics of the brakes.

The brake moves from *Idle* to the *Brake* state when the control input  $u_d(t)$  takes a value greater than 0. When in the *Brake* state, the dynamics of the force generated by the brake is modelled as

$$\begin{aligned} \dot{b}(t) &= \begin{cases} \gamma u_b(t) & \text{if } b(t) < b_{MAX}(t) \\ 0 & \text{otherwise} \end{cases} \\ \dot{b}_{MAX}(t) &= 0. \end{aligned} \quad (6)$$

for some  $\gamma > 0$ . The brake force can be increased by positive values of  $u_b(t)$ . When the force is reduced, the brake moves to the *Release* state. In such a state, the brake keeps applying a force to the train but its value can only decrease. The rate at which the brake reduces its force is constant and, in the proposed model, cannot be controlled. Also, in the *Release* state, the maximum brake force decreases at the same rate as the brake force  $b(t)$ . The dynamics of the brake force in the *Release* state can be modelled as

$$\begin{aligned} \dot{b}(t) &= \begin{cases} -\delta & \text{if } 0 < b(t) \\ 0 & \text{otherwise} \end{cases} \\ \dot{b}_{MAX}(t) &= -\delta, \end{aligned} \quad (7)$$

where  $\delta$  determines the recharging time for the air brake. The brake moves to the *Idle* state only when  $b(t)$  reaches the zero value. Figure 1 shows the transitions among the discrete states and the dynamics of the continuous states.

### 2.3 Combined/System model

As the system described in the previous sections includes continuous-time dynamics and discrete-time events, we can rewrite the system as an impulsive system, using the formalism discussed in [4]. Defining the continuous states as  $\mathbf{x} = (\xi, v, p, b, b_{MAX})^T$ , the joint continuous and discrete states as  $\mathbf{q} = (\mathbf{x}^T, \zeta)^T$  and the inputs as  $\mathbf{u} = (u_b, u_d)^T$ , the combined system can be written as follows:

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} \mathbf{x} \\ \zeta \end{pmatrix} &= \begin{pmatrix} F(\mathbf{x}, \mathbf{u}) \\ 0 \end{pmatrix}, & \text{for } \mathbf{q} \in C, \\ \begin{pmatrix} \mathbf{x}^+ \\ \zeta^+ \end{pmatrix} &= \begin{pmatrix} \mathbf{x} \\ \zeta + 1 \pmod{3} \end{pmatrix}, & \text{for } \mathbf{q} \in D, \end{aligned} \quad (8)$$

where  $C \subseteq \mathbb{R}^5$  defines the flow set and, in virtue of Figure 1, the jump set  $D$  set can be defined as  $D = D_1 \cup D_2 \cup D_3$ , with  $D_1 = \{\mathbf{q} : \zeta(t) = 1, u_b(t) \geq 0\}$ ,  $D_2 = \{\mathbf{q} : \zeta(t) = 2, u_b(t) \leq 0\}$  and  $D_3 = \{\mathbf{q} : \zeta(t) = 3, b(t) = 0\}$ . Notice how the  $D$  set represents the guards in the state machine described in Figure 1, and the  $C$  set defines the operating region of the physical system, where the model herein described is valid. The map  $F$  can be defined from equations (1)-(7). The system flows on  $C$  and experiences a jump on the set  $D$ . In this report we skip discussions on hybrid time domains and the existence and uniqueness of hybrid solutions as it falls outside the scope of this benchmark.

## 3 Control problem and safety requirements

The train control problem deals with the selection of a strategy to control the force generated by the engine and the power generated by the brake. The goal of the control strategy is the minimisation of the energy spent to reach a desired destination while also attempting to match a given speed profile and enforcing the given safety requirements.

The speed profile is defined as a function  $\bar{v}(\xi)$  over the interval  $[0, L]$ , where  $L$  is the total trip distance. In general, a variety of safety requirements must be enforced by ATO, e.g., maximum pull-traction forces among wagons, maximum time that the engine and the brake can be used at their maximum value and maximum and minimum speed of the train along the trajectory. In this report, we focus on the maximum speed of the train over the length of the path which we denote with  $\hat{V}(\xi)$ .

With  $v(\xi; \mathbf{u}, \mathbf{q}(0))$  we denote the speed of the train at the point  $\xi$  when the control action is  $\mathbf{u}$  and the initial state is  $\mathbf{q}(0)$ . The cost associated with  $\mathbf{u}$  can be written as:

$$J(\mathbf{u}, \mathbf{q}(0)) = \int_0^{+\infty} \mathbf{u}(\tau)^\top R \mathbf{u}(\tau) d\tau + \int_0^L \left( v(\rho; \mathbf{u}, \mathbf{q}(0)) - \bar{v}(\rho) \right)^2 d\rho, \quad (9)$$

for some user-selected positive definite matrix  $R$ . The constraints to the control problem are given by the equations modelling train and brake dynamics and by the safety speed constraints. The optimal control problem can now be written as

$$\min_{\|\mathbf{u}\|_\infty \leq 1} J(\mathbf{u}, \mathbf{q}(0)) \quad (10)$$

$$\text{s.t.} \quad (8) \quad (11)$$

$$v(\xi; \mathbf{u}, \mathbf{q}(0)) \leq \hat{V}(\xi). \quad (12)$$

The challenge set by the proposed benchmark lies on both the selection of a suitable control strategy and on the analysis of the enforcement of the safety constraints by the control strategy.

## 4 Implementation

A possible implementation of the train and brake model is given. The models are implemented in Python. The code is composed by 4 files named: `run.me.py`, `brake.py`, `train.py`, `trainpath.py`.

The file `run.me.py` is responsible for configuring the train, the path, and the brake with their parameters, for executing the simulation, and generating the final graphs. The files `brake.py` and `train.py` provide a possible implementation of the air brake and of the train models previously discussed. The file `trainpath.py` can be used to generate test paths for the train.

The proposed Python implementation is provided as an example to facilitate the adoption of the proposed benchmark. The code has been developed for the sole purposes of this workshop. The authors are grateful for any help aimed to improve the code quality.

## 5 Model discussion and possible extensions

The model as discussed in Sections 2 and 3 is presented as a first attempt to model the complex dynamics of a train equipped with an air brake as a hybrid model. In order to make the model more realistic, the authors propose several possible extensions:

- The train is modelled as a point mass. A more realistic approach would be to model each car of the train as a point mass and couple the individual cars by stiffness and damping functions. An even more complex model would describe the train by a distributed mass model.
- With increasing production of natural gas, dual fuel engines become more attractive for trains as well. When a dual fuel engine is modelled correctly, also the train model itself becomes a hybrid system.
- In the proposed models, the same slew rates for all input control values are considered. Different slew rates for the different actuators can be considered.
- The actual force used by the brake is not known exactly. Uncertainties of up to 50% of the nominal value are possible. A possible extension, is therefore to include an uncertainty description to the brake model.

- The model as presented does not consider any delays for the brake actuation. A more realistic approach however, would be to use delays to model how the force propagates along the train. This becomes more relevant when the train is modelled as a distributed-mass system. Additionally, air brakes cannot be used continuously in reality for a long time, since they heat up and lose efficiency. To model these constraints, bounds on the maximum time the brake is used can be added to the model.

In addition to the model extensions discussed previously, the control problem itself can also be made more complex by adding additional optimisation criteria such as maximum pull-tracking forces among the wagons or other requirements discussed in Section 3.

## 6 Acknowledgements

The authors gratefully acknowledge financial support by the European Commission project UnCoVerCPS under grant number 643921. The authors would like to thank also Kirk Mathews and Ramu Chandra for the fruitful discussions on the topic.

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