Some Game via Grill-Semi-P-Open

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ABSTRACT

This research presents some kind of games through open collection Ǥ-spo set using the grill topological space that are games of type which is Game (Ǥ-SP-$\mathcal{T}_i - space$, Ǥ) , when i={0,1,2} By using many figures and proposition, the relation between these types of games has been studied with explaining some examples.

Keywords. Game g (Ǥ-SP-$\mathcal{T}_0$-$space$, Ǥ), Game g(Ǥ-SP-T_1-space , Ǥ), Game g(Ǥ-SP-T_2-space , Ǥ).

Introduction

A nonempty family G $of\ a\ topological\ space$ Ẋ is named $a\ Grill$ whenever
i. M ∈ Ǥ and M ⊆ S ⊆ Ẋ then S ∈ Ǥ.
ii. M, S ⊆ Ẋ ∧ M ∪ S ∈ Ǥ then M ∈ Ǥ ∨ S ∈ Ǥ. [1] Suppose that Ẋ is a nonempty set, Then the following families are grills on Ẋ. [1-3]

1) ∅ and p(Ẋ) \ {∅} are trivial examples of a grill on Ẋ
2) $Ǥ_\infty$ which is the collection of all infinite subsets of Ẋ.
3) $Ǥ_{co}$ which is the collection of all uncountable subsets of Ẋ.
4) $Ǥ_p$= {Λ: Λ∈ p(Ẋ), p ⊆ Λ} is a specific point grill on Ẋ.
5) $Ǥ_Λ$= {S: S∈p(Ẋ), S∩M ≠ ∅ }, and If (Ẋ,$\mathcal{T}$) is a topological space, then the family of all non-nowhere dense subsets called Ǥ= {M:$int_{\mathcal{T}}\ cl_{\mathcal{T}}(M) \neq \emptyset$ }. Is the one of kinds of a grill on Ẋ. Suppose that Ǥ is a grill on (Ẋ,$\mathcal{T}$) The operator Ǿ: p(Ẋ)→p(Ẋ) is defined by Ǿ (M)={x ∈ Ẋ\ ů ∩ M ∈ Ǥ, $for\ all$ ů ∈ $\mathcal{T}$(Ẋ)},$\mathcal{T}$(Ẋ) indicate the neighborhood of x. A mapping Ψ: p(Ẋ)→p(Ẋ) is defined as Ψ (M) = M ∪ Ǿ (M) for all M ∈ p(Ẋ). [4,5]

$The\ map$ Ψ satisfies Kuratowski closure axioms: [3,4]
1. Ψ(∅) = ∅
2. If M ⊆S, then Ψ (M) ⊆ Ψ(S),
3. If M ⊆ Ẋ, then Ψ (Ψ (M)) =Ψ (M),
4. IfM,S ⊆ Ẋ, then Ψ (M ∪ S) =Ψ (M) ∪ Ψ (S).

$A\ subset$ M of (Ẋ,$\mathcal{T}$) $is\ a$ preopen set if M ⊆ intcl M $The\ complement\ of\ a$ preopen $set\ is$ named preclosed set. The collection of all preopen sets of Ẋ is indicate by po(Ẋ).$The\ collection\ of\ all$ preclosed sets of Ẋ is indicate by pc(Ẋ). [7]
Now PCL=∩ {M ⊆ Ẋ ; ů ⊆ M whenever M^c ∈ PO(Ẋ)}. [7]

$A\ subset$ M of (Ẋ, $\mathcal{T}$) is named semi-p-open set, if and only $if\ there\ exists\ a$ preopen set in Ẋ say Ụ such that Ụ ⊆ M ⊆ PCL Ụ. $The\ collection\ of\ all$ semi-p-open $sets\ of$ Ẋ is indicated by S-PO(Ẋ). The complement of a semi-p-closed set. The family of all semi-p-closed sets of Ẋ is indicated by S-PC(Ẋ). [7]

It is clearly that every preopen set is a S-PO set. [7]

In this paper, we study the G-SP-closed set and its complement G-SP-open set with many functions by these notions like: G-SP-$open$ function, G*-SP-open function, G**-SP-open function, G-SP-continuous, strongly-G-SP-continuous, and G-SP-irresolute function .and new separation axioms like G-SP-$\mathcal{T}_0$ -space, G-SP-$\mathcal{T}_1$-space, G-SP-$\mathcal{T}_2$-space with G-SP-convergence sequence.

Let g be a game between two players ρ_1 and ρ_2. The set of $choices$ $Ĺ_1, Ĺ_2, Ĺ_3, \ldots, Ĺ_n$ For each player. These choices are called moves or options. [6,7]

We have two kinds of games are alternating game, and simultaneous game

which will be explained in the following. Alternating game is one of players ρ_1 chose one of the $options$ $Ĺ_1, Ĺ_2, Ĺ_3, \ldots, Ĺ_n$ .Next player ρ_2 choose one of these moves when knowing the chooses of ρ_1. In alternating games must determine the player who he starts the game [8,9]. A simultaneous game is both players select their moves in the same time without knowing the choice of the other player. If a game has more than one stage then the game is called a repeated game. The game possible infinite or finite to the number repetition of the game in the end of all stage all players get a certain reward [6].

In this research provided the sorts of game through a given set. The winning and losing strategy for any player ρ in the game g, if ρ has a winning strategy in gshortly by ($\rho \nearrow g$) and if P does not have a winning strategy shortly by (ρ ⇗ g), if P has a losing strategy shortly by (ρ ↘ g) and if ρ does not has a losingstrategy shortly by (ρ ⇘ g).

1.Preliminaries.

Definition 1. 1: [10] Let (Ẋ, $\mathcal{T}$) be a topological space, define a Game g (T_0, Ẋ) as follows: The two players ρ_1 and ρ_2 are play an inning for each natural numbers, in the Z- th inning, the first round, ρ_1 will choose $m_z \neq s_z$, when $m_z, s_z \in$ Ẋ.

Next, ρ_2 choose $V_z \in \mathcal{T}$ such that $m_z \in V_m$ and $s_z \notin V_z$, ρ_2 wins in the game,when $\beta = \{ V_1, V_2, V_3, \ldots, V_z, \ldots \}$ satisfies that for all $m_z \neq m_z$ in Ẋ there exist β such that $m_z \in V_z$ and $s_z \notin V_z$. Other hand ρ_1 wins.

Definition 1. 2: [10] Let (Ẋ, $\mathcal{T}$) be a topological space, define a Game g ($\mathcal{T}_1$,Ẋ) as follows: The two players ρ_1 and ρ_2 are play an inning for each natural numbers, in the Z- th inning, the first round, ρ_1 will choose $m_z \neq s_z$, when $m_z, s_z \in$ Ẋ.

Next, ρ_2 choose $V_z, Q_z \in \mathcal{T}$ such that $m_z \in (V_z - Q_z)$ and $s_z \in (Q_z$- $V_z)$, ρ_2 wins in the game, when β = {{ V_1, Q_1}, { V_2, Q_2}, …., { V_z, Q_z}, ….} $h_s \in (Q_z$- $V_z)$, satisfies that for all $m_z \neq s_z$ in Ẋ there exists { V_z, Q_z } $\in \beta$ such that $m_z \in (V_z - Q_z)$ and $s_z \in (Q_z$- $V_z)$. Other hand ρ_1 wins.

Definition 1. 3: [10] Let $(\dot{X}, \mathcal{T})$ be a topological space, define a Game g $(\mathcal{T}_2, \dot{X})$ as follows: The two players ρ_1 and ρ_2 are play an inning for each natural numbers, in the z- th inning, the first round, ρ_1 will choose $m_z \neq s_z$, where $m_z, s_z \in \dot{X}$.

Next, ρ_2 choose V_z, Q_z are disjoint, $V_z, Q_z \in \mathcal{T}$ such that $m_z \in V_z$ and $s_z \in Q_z$, ρ_2 wins in the game, where β ={{ $V_1 Q_1$}, { V_2, Q_2}, …., { V_z, Q_z}, …. } , satisfies that for all $m_z \neq s_z$ in $\dot{X}$ there exists { V_z, Q_z } $\in \beta$ such that $m_z \in V_z$ *and* $s_z \in Q_z$. Other hand ρ_1 wins.

2.𝔾-SP-Openness on Game.

Definition 2.1: *Let* $(\dot{X}, \mathcal{T}, \mathbb{G})$ *be a grill topological space and let* M ⊆ $\dot{X}$, Then M is called Grill semi-p-open set denoted by "G-SPO set " if ∃ v ∈ PO($\dot{X}$) such that v-M ∉ G ∧ M-PCL(v) ∉G. the set of all G-SPO sets denoted by G-SPO($\dot{X}$).

Example2.2: Let $\dot{X}$= $\{m_1, m_2, m_3\}$, $\mathcal{T}$= {$\dot{X}$,∅,$\{m_1\}$}

PO($\dot{X}$)= {ů ⊆ $\dot{X}$; m_1 ∈ ů} ∪ ∅ , PC($\dot{X}$) = {$\mathcal{F}$ ⊆ $\dot{X}$; $m_1 \notin \mathcal{F}$ } ∪ $\dot{X}$.

Then G-SPO ($\dot{X}$) = $p(\dot{X})$.

Example 2.3: *Let* $\dot{X} = \{m_1, m_2, m_3, m_4\}, \mathcal{T} = \{\dot{X}, \emptyset, \{m_1\}, \{m_4\}, \{m_1, m_4\}\}$,G= p($\dot{X}$) \ {∅},

PO ($\dot{X}$) = $\{\dot{X}, \emptyset, \{m_1\}, \{m_4\}, \{m_1, m_4\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}\}$.

PC($\dot{X}$) = $\{\dot{X}, \emptyset, \{m_2, m_3, m_4\}, \{m_1, m_2, m_3\}, \{m_2, m_3\}, \{m_3\}, \{m_2\}\}$, G-$SPO$($\dot{X}$)={$\dot{X}$, ∅, $\{m_1\}, \{m_4\}$, $\{m_1, m_2\}, \{m_1, m_3\}$, $\{m_1, m_4\}, \{m_2, m_4\}, \{m_3, m_4\}$, $\{ m_1, m_2, m_3\}$, $\{m_1, m_2, m_4\}, \{m_2, m_3, m_4\}, \{m_1, m_3, m_4\}$.

Remark 2.4: [7] $\bigcup_{i \in \wedge} PCL(ů_i) \subseteq PCL(\bigcup_{i \in \wedge} ů_i)$.

Proposition 2.5: If M_i ∈ G-SPO($\dot{X}$) ∀ i ∈ ∧, *then* $\bigcup_{i \in \wedge} M_i$ ∈ G-SPO($\dot{X}$).

Proof: Let M_i ∈ G-SPO($\dot{X}$), ∃ ů ∈ PO($\dot{X}$), ($ů_i - M_i$) ∉G ∧(M_i −PCL($ů_i$))∉ G∀ i ∈∧ . this implies, $\bigcup_i$($ů_i$- M_i) ∉ G,*so* $(\bigcup_i ů_i - \bigcup_i M_i) \subseteq \bigcup_i(ů_i - M_i)$ ∉ G, therefore, $(\bigcup_i ů_i - \bigcup_i M_i)$ ∉ G,

On the other hands, (M_i − PCL ($ů_i$)) ∉ G ∀ i ∈ ∧ ,$\bigcup_i \left(M_i - PCL(ů_i)\right)$ ∉ G,

$(\bigcup_i M_i - \bigcup_i PCL\ (ů_i)) \subseteq \bigcup_i\left(M_i - PCL(ů_i)\right)$ ∉ G so, $\bigcup_i M_i - \bigcup_i(PCL(ů_i))$ ∉ G ,since $\bigcup_i PCL(ů_i) \subseteq PCL(\bigcup_I ů_i)$, there for ($\bigcup_{i \in \wedge} M_i - PCL(\bigcup_i ů_i)) \subseteq (\bigcup_i M_i - \bigcup_i PCL(ů_i))$ ∉ G so,$(\bigcup_i M_i - PCL(\bigcup_i ů_i))$ ∉ G.

Corollary 2.6: If $\mathcal{F}_i$ ∈ G-SPC($\dot{X}$), then $\bigcap_i \mathcal{F}_i$ ∈ G-SPC($\dot{X}$).

Remark 2.7: *let* M, S ∈ G -SPO($\dot{X}$) then $M \cap S$ need not to be a G-SPO set.

Example 2.8: *Let* $\dot{X} = \{m_1, m_2, m_3, m_4\}, \mathcal{T} = \{\dot{X}, \emptyset, \{m_1\}, \{m_4\}, \{m_1, m_4\}\}$,

$PO(\dot{X}) = \left\{\dot{X}, \emptyset, \{ m_1 \}, \{m_4\}, \{m_1, m_4\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}\right\}$,

$PC(\dot{X}) = \{\dot{X}, \emptyset, \{m_2, m_3, m_4\}, \{m_1, m_2, m_3\}, \{m_2, m_3\}, \{m_3\}, \{m_2\}\}$, G=p($\dot{X}$)\{∅},

G-$SPO(\dot{X}) = \{\dot{X}, \emptyset , \{m_1\}, \{m_4\}, \{m_1, m_2\}, \{m_1, m_3\}, \{m_1, m_4\}$,

$\{m_2, m_4\}, \{m_3, m_4\}, \{m_1, m_2, m_3\}, \{m_1, m_2, m_4\}, \{m_2, m_3, m_4\}, \{m_1, m_3, m_4\}\}$,

Remark 2.9: let M , S ∈ G -SPC($\dot{X}$) then M ∪ S need not be a G-SPC set.

See Example 2.8, *let* $M = \{m_1, m_2, m_3\}$, $S = \{m_2, m_4\}$, $M^c = \{m_4\}$, $S^c = \{m_2, m_3\}$,M^c ,S^c *are* G-$SPC(\dot{X})$, then $M^c \cup S^c = \{m_1, m_3, m_4\}$ which is not a G-SPC($\dot{X}$).

Remark 2.10: [7] Each open set is a preopen set.

Proposition 2.11: Each open set is a G -SPO set.

Proof: Let $M \in \mathcal{T}$ by Remark 2.4, so M is a preopen set; $\exists M \in po(\dot{X})$, such that, M-M = $\{\emptyset\} \notin$ Ǥ, And M-PCL $(M) = \{\emptyset\} \notin$ Ǥ, $therefor\ M$ is a Ǥ-SPO set.
Corollary2.12: If F is a closed set, then F is a Ǥ-SPC set.
Proposition 2.13: Every semi-PO set is Ǥ-SPO set.
Proof: Let $M \in$ S-PO(Ẋ) for that $\exists$ ů $\in PO(\dot{X})$ $such\ that$ ů $\subseteq$ M $\subset PCL$(M), further more ů-M= $\{\emptyset\} \notin$ Ǥ $\wedge$ M-$PCL(M) = \{\emptyset\} \notin$ Ǥ. Hence, M is a Ǥ-SPO set.
As for the reverse proposition (2.13) it is not necessarily to be achieved.
Example 2.14: suppose that
$\dot{X} = \{m_1, m_2, m_3, m_4\}, \mathcal{T} = \{\dot{X}, \emptyset, \{m_1\}, \{m_4\}, \{m_1, m_4\}\}$,
Ǥ= $\emptyset$,$PO(\dot{X}) = \{\dot{X}, \emptyset, \{m_1\}, \{m_4\}, \{m_1, m_4\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}\}$,
$PC(\dot{X}) = \{\dot{X}, \emptyset, \{m_2, m_3, m_4\}, \{m_1, m_2, m_3\}, \{m_2, m_3\}, \{m_3\}, \{m_2\}\}$,
Ǥ-SPO(Ẋ)=p(Ẋ). Then $\{m_2\} \in$ Ǥ-$SPO(\dot{X})$,But $\{m_2\} \notin$ Ǥ -SPO(Ẋ).

Remark2.15: The collection of all Ǥ-SPO set is a supra topological space.

__**Definition 2.16:**__ The space $(\dot{X}, \mathcal{T}, Ǥ)$ is a Ǥ-SP-$\mathcal{T}_0$-$space\ \ if\ for\ each\ M \neq S$ and M ,$S \in \dot{X}$,$there$ exist Ų ∈ Ǥ-SPO(Ẋ) whenever, $M \in$ Ų $and\ S \notin$ Ų or M $\notin$ Ų and S ∈ Ų.

__**Definition 2.17:**__ The space (Ẋ, $\mathcal{T}$, Ǥ) is a Ǥ-SP-T_1-space if for each M $\neq$ S and M , S ∈ Ẋ. then there exist V_1, V_2 are Ǥ-SPO set, whenever $M \in V_1$, $S \notin V_1$ and M $\in V_2$,$S \notin V_2$.

__**Definition 2.18:**__ The space (Ẋ, $\mathcal{T}$, Ǥ) is a Ǥ-SP-T_2-space if for each M $\neq$ S , there are Ǥ-SPO set $\exists$ V_1, V_2 wherever M $\in V_1$, S $\in V_2$ and $V_1 \cap V_2$ = { $\emptyset$} .
__**Remark 2.19:**__ If The space (Ẋ,$\mathcal{T}$, Ǥ) is a Ǥ-SP-T_{i+1}-space then it is a Ǥ-SP-T_i-space (for every i ∈ {0,1}).

3-Some Game in Ǥ-SP-open sets

__**Definition 3.1:**__
Let (Ẋ, $\mathcal{T}$, Ǥ) be a grill topological space, define a Game g (Ǥ-SP-$\mathcal{T}_0$, Ẋ) as follows: the two players ρ_1 and ρ_2 are play an inning for each natural numbers, in the Z- th inning, the first round, ρ_1 will choose $m_z \neq s_z$, when $m_z, s_z \in \dot{X}$. Next , ρ_2 choose $V_z \in$ Ǥ-$SPO(\dot{X})$ such that $m_z \in V_z$ and $s_z \notin V_z$, ρ_2 wins in the game, when β = { $V_1, V_2, V_3, \dots, V_z, \dots$ } satisfies that for all $m_z \neq s_z$ in Ẋ there exist β such that $m_z \in V_z$ and $s_z \notin V_z$. Other hand ρ_1 wins.
__**Example 3.2:**__
Let Game g($\mathcal{T}_0$,Ẋ) be a game, Ẋ= $\{m_1, m_2, m_3, m_4\}$and $\mathcal{T} = \{\dot{X}, \emptyset, \{m_1\}, \{m_4\}, \{m_1, m_4\}\}$.
Ǥ=$\emptyset$ therefor, Ǥ-SPO(Ẋ)= P(Ẋ) then in the first round ρ_1 will choose $m_1 \neq m_2$, whenever $m_1, m_2 \in$ Ẋ.Next, ρ_2 $choose$ $\{m_1\} \in$ Ǥ $- SPO(\dot{X})$ $such\ that$ $m_1 \in \{m_1\}$ and $m_2 \notin \{m_1\}$, in the second round ρ_1 $will\ choose$ $m_1 \neq m_2$, whenever $m_1, m_2 \in$ Ẋ. Next $\rho_2$choose $\{m_1\} \in$ Ǥ $- SPO(\dot{X})$ Such that $m_1 \in \{m_1\}$ and $m_2 \notin \{m_1\}$,in the third round ρ_1 $will\ choose$ $m_2 \neq m_3$ whenever $m_2, m_3 \in$ Ẋ . Next, $\rho_2$choose $\{m_3\} \in$ Ǥ $- SPO(\dot{X})$ Such that $m_2 \in \{m_2\}$ and $m_3 \notin \{m_2\}$, $\rho_2$wins in the game, whenever, $\beta = \{\{m_1\}, \{m_2\}\}$.
__**Remark 3.3:**__ for any grill topological space (Ẋ, $\mathcal{T}$, Ǥ) :

i. If $\rho_2 \nearrow$ in a Game g ($\mathcal{T}_0$, Ẋ) then $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ).
ii. If $\rho_2 \searrow$ in a Game g ($\mathcal{T}_0$, Ẋ) then $\rho_2 \searrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ).
iii. If $\rho_1 \nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ) then $\rho_1 \nearrow$ in a Game g ($\mathcal{T}_0$, Ẋ).

Proof: It is clear by Remark 2.4.

Theorem3.4 Let (Ẋ , $\mathcal{T}$, G) is a G-SP- $\mathcal{T}_0$- space if and only if $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ)

Proof: Since (Ẋ, $\mathcal{T}$, G) is a G-SP- $\mathcal{T}_0$- space then in the Z- th inning any choice for the first player ρ_1 choose $m_z \neq s_z$, when $m_z, s_z \in$ Ẋ. The second player ρ_2 can be found $V_z \in$ G-SPO(Ẋ) .thus $\beta = \{ V_1, V_2, V_3, \dots, V_z, \dots \}$ is the winning strategy for ρ_2.

Conversely, Clear.

Theorem 3. 5: The grill topological space (Ẋ, $\mathcal{T}$, G) is a G-SP-T_0-space if and only if for each elements m ≠ S there exists two G-SPC set containing only one of them.

***Proof*:** ⟹) Let m and S are two distinct elements in Ẋ. Since Ẋ is a G-SP-T_0-space then there is a G-SPO set V containing only one of them, then (Ẋ-V) is a G-SPC sets containing the other one.

⟸)Conversely, let m and S are two distinct elements in Ẋ and there is a G-SPC set W containing only one of them. then (Ẋ- W) is a G-SPO set containing the other one.

Corollary 3.6: For a space (Ẋ,$\mathcal{T}$, G), $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ) if and only if, for every $s_1 \neq s_2$ in Ẋ, there exists V ∈ G-SPC(Ẋ) such that $s_1 \in V$ and $s_2 \notin V$.

Proof: Let $s_1, s_2 \in$ Ẋ when $s_1 \neq s_2$, since $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ) then by Theorem 3.4, the space (Ẋ , $\mathcal{T}$, G) is a G-SP- $\mathcal{T}_0$- space. Then Theorem 3.5 is applicable.
Conversely, By Theorem 3.5 the grill topological space (Ẋ , $\mathcal{T}$, G) is a G-SP- $\mathcal{T}_0$- space Then Theorem 3.4 is applicable.

Corollary 3.7: let (Ẋ, $\mathcal{T}$, G) is a G-SP- $\mathcal{T}_0$- space if and only if $\rho_1 \not\nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ).

Proof: By theorem 3.4 the proof is over.

Theorem 3.8 : let (Ẋ, $\mathcal{T}$, G) is not a G-SP- $\mathcal{T}_0$- space if and only if $\rho_1 \nearrow$ in a Game g(G-SP-$\mathcal{T}_0$, Ẋ) .

Proof: In the Z-th inning ρ_1 in a Game g (G-SP-$\mathcal{T}_0$, Ẋ) choose $m_z \neq s_z$, where $m_z, s_z \in$ Ẋ. ρ_2 in a Game g (G-SP-$\mathcal{T}_0$, Ẋ) cannot be found V_z is a G-SPO sets containing only one element of them, because (Ẋ,$\mathcal{T}$, G) is not G -SP-$\mathcal{T}_0$-space hence $\rho_1 \nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ).

Conversely, Clear.

Corollary 3.9: let (Ẋ, $\mathcal{T}$, G) be not a G-SP- $\mathcal{T}_0$- space if and only if $\rho_2 \not\nearrow$ in a Game g (G-SP-$\mathcal{T}_0$, Ẋ).

Proof: By theorem 3.4 the proof is over.

Definition 3. 10: Let (Ẋ,$\mathcal{T}$, G) be a grill topological space, define a Game g (G-SP-$\mathcal{T}_1$, Ẋ) as follows: The two players ρ_1 and ρ_2 are play an inning for each natural numbers, in the Z- th inning, the first round, ρ_1 will choose $m_z \neq s_z$, where $m_z, s_z \in$ Ẋ. Next , ρ_2 choose $V_m , V_s \in$G-SPO(Ẋ) such that $m_z \in (V_m - V_S)$ and $s_z \notin (V_S - V_m)$, ρ_2 wins ,P$_2$ wins in the game, where , β ={{ $V_1 V_1$}, { V_2, V_2}, …., { V_z, V_z}, …. } ,

satisfies that for all $m_z \neq s_z$, in Ẋ there exists{ V_m, V_S} $\in \beta$ such that $m_z \in (V_m - V_S$ and $s_z \in (V_S - V_m)$. Other hand ρ_1 wins.

Example 3.11:
Let Game g(Ẋ,$\mathcal{T}$, G) be a game, Ẋ= $\{m_1, m_2, m_3\}$and $\mathcal{T} = \{$Ẋ$, \emptyset, \{m_2\}\}$.
G={û⊆ Ẋ; $m_2 \in$ û}, PO(Ẋ)= {Ẋ,$\emptyset$, $\{m_2\}, \{m_2, m_3\}, \{m_1, m_2\}$} and
PC(Ẋ)= {Ẋ, $\emptyset$, $\{m_1, m_3\}, \{m_1\}, \{m_3\}$}therefor, G-SPO(Ẋ)= {Ẋ,$\emptyset$, $\{m_2\}, \{m_1, m_2\}, \{m_2, m_3\}$}
then in the first round ρ_1 will choose $m_1 \neq m_2$, whenever
$m_1, m_2 \in$ Ẋ.Next ρ_2 *connot be found*$V_z, Q_z \in$ G $- SPO$(Ẋ) such that $m_1 \in (V_z - Q_z)$and $m_2 \in (Q_z - V_z)$, so ρ_1 wins in the game.

Remark 3. 12: For any grill topological space (Ẋ,$\mathcal{T}$, G):

i. If $\rho_2 \nearrow$ in a Game g ($\mathcal{T}_1$, Ẋ) then $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ).

ii. If $\rho_2 \searrow$ in a Game g ($\mathcal{T}_1$, Ẋ) then $\rho_2 \searrow$ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ).

iii. If $\rho_1 \nearrow$ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ) then $\rho_1 \nearrow$ in a Game g ($\mathcal{T}_1$, Ẋ).

Theorem 3. 13: Let (Ẋ,$\mathcal{T}$, G) is a G-SP - space if and only if $\rho_2 \nearrow$ a Game g(G-SP-$\mathcal{T}_1$,Ẋ).

Proof: Let (Ẋ,$\mathcal{T}$, G) be a grill topological space, in the first round ρ_1 will choose $m_1 \neq s_1$, where $m_1, s_1 \in$ Ẋ. Next, since (Ẋ,$\mathcal{T}$, G) is a G-SP - space ρ_2 can be found $V_1, Q_1 \in$ G-SP O(X) such that $m_1 \in (V_1 - Q_1)$ and $s_1 \in (Q_1 - V_1)$ in the second round ρ_1 will choose $m_2 \neq s_2$, where $m, s_2 \in$ Ẋ. Next, ρ_2 can be found $V_2, Q_2 \in$ G-SP O(X) such that $m_2 \in (V_2 - Q_2)$ and $s_2 \in (Q_2 - V_2)$, in the Z- th round, ρ_1 will choose $m_n \neq s_n$, where$m_n, s_n \in$ Ẋ. Next, ρ_2 can be found $V_n, Q_n \in$ G-SP O(X) such that $m_n \in (V_n - Q_n)$ and $s_n \in (Q_n - V_n)$. Thus β= {{ V_1, Q_1}, { V_2, Q_2}, …, { V_n, Q_n}, …} is the winning strategy forρ_2 .

Conversely Clear.

Theorem 3. 14: The grill topological space (Ẋ, $\mathcal{T}$, G) is a G-SP-T_1 -space if and only if for each elements m $\neq$ s there exists two G-SPC sets H_1 and H_2 such that m $\in (H_1 - H_2)$ and s $\in$

$(H_2 - H_1)$.

***Proof*:** Let m and s are two distinct elements in Ẋ. Since Ẋ is a G-SP-T_1-space then there exists two G-SPO sets H_1 and H_2 such that m ∈ $(H_1 - H_2)$ and s ∈ $(H_2 - H_1)$. then there exists G-SPC sets (Ẋ-V_1) and (Ẋ- V_2) such that m ∈ ((Ẋ- H_2)- (Ẋ- H_1)), s ∈ ((Ẋ- H_1) - (Ẋ- H_2)) where (Ẋ- H_1) = Ӱ1 and (Ẋ- H_1) = Ӱ2. then there exists two G-SPC sets Ӱ1 and Ӱ2 satisfy x ∈ (Ӱ1 - Ӱ2^c) and s∈ (Ӱ2 -Ӱ1^c)there for x ∈ (Ӱ1 - Ӱ2) and s ∈ (Ӱ2 - Ӱ1).

Conversely Let m and s are two distinct elements in Ẋ and there exists two G-SPC sets Ӱ1 and Ӱ2 satisfy m ∈ (Ӱ1 - Ӱ2^c) and s ∈ (Ӱ2-Ӱ1^c) then there exists G-SPO set (Ẋ - Ӱ1) and (Ẋ - Ӱ2) whenever m ∈ ((Ẋ - Ӱ2) - (Ẋ - Ӱ1)), s ∈ ((Ẋ -Ӱ1) - (Ẋ - Ӱ2)) where (Ẋ - Ӱ2) = H_1 and (Ẋ - Ӱ1) = H_2 .

<u>Corollary 3.15:</u> For a space (Ẋ,$\mathcal{T}$, Ǥ), ρ_2 ↗ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ) if and only if, for every $s_1 \neq s_2$ in Ẋ, there exists Ӱ1, Ӱ2 ∈ G-SPC(X) such that s_1 ∈ (Ӱ1 - Ӱ2) and s_2 ∈ (Ӱ2 - Ӱ1).

Proof*:* Let $s_1 \neq s_2$ where s_1, s_2 ∈ Ẋ, since ρ_2 ↗ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ) then by Theorem 3.13, the space (Ẋ,$\mathcal{T}$, Ǥ) is G-SP-$\mathcal{T}_1$-space. Then Theorem 3.14 is applicable.

Conversely, by Theorem 3.14 the grill topological space (Ẋ,$\mathcal{T}$, Ǥ) is a G-SP-$\mathcal{T}_1$ -space Then Theorem 3.13 is applicable.

<u>Corollary 3. 16:</u> Let (Ẋ,$\mathcal{T}$, Ǥ) is a G-SP-$\mathcal{T}_1$ - space if and only if ρ_1 ⇗ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ)

Proof: By Theorem 3.13, the proof is over.

<u>Proposition 3. 17:</u> Let (Ẋ,$\mathcal{T}$, Ǥ) is not G-SP- T_1- space if and only if ρ_1 ↗ in a Game g (G-SP-$\mathcal{T}_1$, Ẋ).

Proof: In the Z-th inning ρ_1 in a Game g(G-SP-$\mathcal{T}_1$, Ẋ) choose $x_m \neq x_s$, where x_m , x_s ∈ Ẋ , ρ_2 a Game g(G-SP-$\mathcal{T}_1$, Ẋ), cannot be found V_m, Q_m are two G-SPO sets such that x_m ∈ (V_m -Q_m) and x_s ∈ (Q_m -V_m) because (Ẋ,$\mathcal{T}$, Ǥ) is not G-SP-T_1-space henceρ_1 ↗ a Game g(G-SP-$\mathcal{T}_1$, Ẋ)

Conversely Clear.

<u>Corollary 3.18:</u> Let (Ẋ,$\mathcal{T}$, Ǥ) is not G-SP- $\mathcal{T}_1$-space if and only if ρ_2 ⇗ in a Game g(G-SP-$\mathcal{T}_1$,Ẋ).

Proof: By theorem 2. 19. The proof is over.

Definition 3.19: Let $(\dot{X},\mathcal{T},Ǥ)$ be a grill topological space define $in\ a$ Game g(Ǥ-SP-$\mathcal{T}_2$,$\dot{X}$) as follows: The two players ρ_1 and ρ_2 are play an inning for each natural number, in the Z- th inning, the first round, ρ_1 will choose $m_z \neq s_z$, where $m_z, s_z \in \dot{X}$. Next, ρ_2 choose V_z, Q_z are disjoint, $V_z, Q_z \in$ Ǥ-SPO($\dot{X}$) such that $m_z \in V_z$ and $s_z \in Q_z$. ρ_2 wins in the game, when $\beta = \{\{V_1, Q_1\}, \{V_2, Q_2\}, \dots, \{V_n, Q_n\}, \dots\}$

satisfies that for all $m_z \neq s_z$ in X there exists $\{V_z, Q_z\} \in \beta$ such that $m_z \in V_z$ and $s_z \in Q_z$. Other hand ρ_1 wins.

By the same way of Example 2. 12 we can be explained that ρ_2 wins in the Game g(Ǥ-SP$\mathcal{T}_2$,$\dot{X}$) where V, Q are two disjoint, Ǥ-SPO sets and β be a collection of all disjoint Ǥ-SPO sets in $\dot{X}$ other hand ρ_1 wins.

Example3.20:

Let Game g($\mathcal{T}_2$,$\dot{X}$) be game ,$\dot{X}=\{m_1, m_2, m_3\}$and $\mathcal{T} = \{\dot{X}, \emptyset, \{m_2\}\}$. Ǥ={û⊆ $\dot{X}$; $m_2 \in$ û} , $PO(\dot{X})=\{\dot{X},\emptyset,\{m_2\},\{m_2,m_3\},\{m_1,m_2\}\}$ and PC($\dot{X}$)=$\{\dot{X},\emptyset,\{m_1,m_3\},\{m_1\},\{m_3\}\}$therefor,Ǥ-SPO($\dot{X}$)= $\{\dot{X},\emptyset,\{m_2\},\{m_1,m_2\},\{m_2,m_3\}\}$ then in the first round ρ_1 will choose $m_1 \neq m_2$, whenever $m_1, m_2 \in \dot{X}$. Next, $\rho_2$cannot be found $V_n, Q_n \in Ǥ - SPO(\dot{X})\ such\ that\ m_1 \in V_n and\ m_2 \in Q_n$, $V_n \cap Q_n = \{\emptyset\}\ thus\ \rho_1$ wins in the game.

Remark 3.21: for any grill topological space $(\dot{X}, \mathcal{T}, Ǥ)$:

i. If $\rho_2 \nearrow$ a Game g ($\mathcal{T}_2$, $\dot{X}$) then $\rho_2 \nearrow$ in a Game g (Ǥ-SP-$\mathcal{T}_2$, $\dot{X}$).
ii. If $\rho_2 \searrow$ a Game g ($\mathcal{T}_2$, $\dot{X}$) then $\rho_2 \searrow$ in a Game g (Ǥ-SP-$\mathcal{T}_2$, $\dot{X}$).
iii. If $\rho_1 \nearrow$ a Game g (Ǥ-SP-$\mathcal{T}_2$, $\dot{X}$) then $\rho_1 \nearrow$ in a Game g ($\mathcal{T}_2$, $\dot{X}$).

Proof: It is clear by proposition 2.4

Theorem 3. 22: A space $(\dot{X},\mathcal{T},Ǥ)$ is a Ǥ-SP-$\mathcal{T}_2$-space if and only if $\rho_2 \nearrow$ in a Game g(Ǥ-SP-$\mathcal{T}_2$,$\dot{X}$).

Proof: Let $(\dot{X}, \mathcal{T}, Ǥ)$ be a grill topological space in the first round ρ_1 will choose $m_1 \neq$ ş$_1$, where $m_1, s_1 \in \dot{X}$. Next since $(\dot{X},\mathcal{T}, Ǥ)$ is a Ǥ-SP-$\mathcal{T}_2$-space ρ_2 can be found V_1 and $Q_1 \in$ Ǥ-SPO($\dot{X}$) such that $m_1 \in V_1\ and\ s_1 \in Q_1$, $V_1 \cap Q_1 = \{\emptyset\}$ in the second round ρ_1 will choose where $m_2 \neq$ ş$_2 \in \dot{X}$. Next choose V_2 and $Q_2 \in$Ǥ-SPO($\dot{X}$) such that $s_2 \in V_2$ and $s_2 \in Q_2$, $V_2 \cap Q_2 = \{\emptyset\}$ in the Z–th round ρ_1 will choose. where $m_n \neq s_n$, where $m_n, s_n \in \dot{X}$. Next ρ_2 choose $V_n, Q_n \in$ Ǥ-SPO($\dot{X}$) such that $m_n \in V_n$ and $s_n \in Q_n$, $V_n \cap Q_n = \{\emptyset\}$. Thus $\beta = \{\{V_1, Q_1\}, \{V_2, Q_2\}, \dots, \{V_n, Q_n\}, \dots\}$ is the winning strategy for P_2.

Conversely Clear.

Corollary 3. 23: A space $(\dot{X}, \mathcal{T}, G)$ is a G-SP-$\mathcal{T}_2$ -space if and only if $\rho_2 \nearrow\!\!\!\!/$ a Game g (G-SP-$\mathcal{T}_2$-$space$, G).

Proof: By Theorem 2. 24 the proof is over.

Theorem 3. 24: A space $(\dot{X}, \mathcal{T}, G)$ is not G-SP-$\mathcal{T}_2$ -space if and only if $\rho_1 \nearrow$ in a Game g (G-SP-$\mathcal{T}_2$, $\dot{X}$).

Proof: By corollary 2. 25 the proof is over.

Corollary 3. 25: A space $(\dot{X}, \mathcal{T}, G)$ is not G-SP-$\mathcal{T}_2$-space if and only if $\rho_2 \nearrow\!\!\!\!/$ In a Game g (G-SP-$\mathcal{T}_2$, $\dot{X}$).

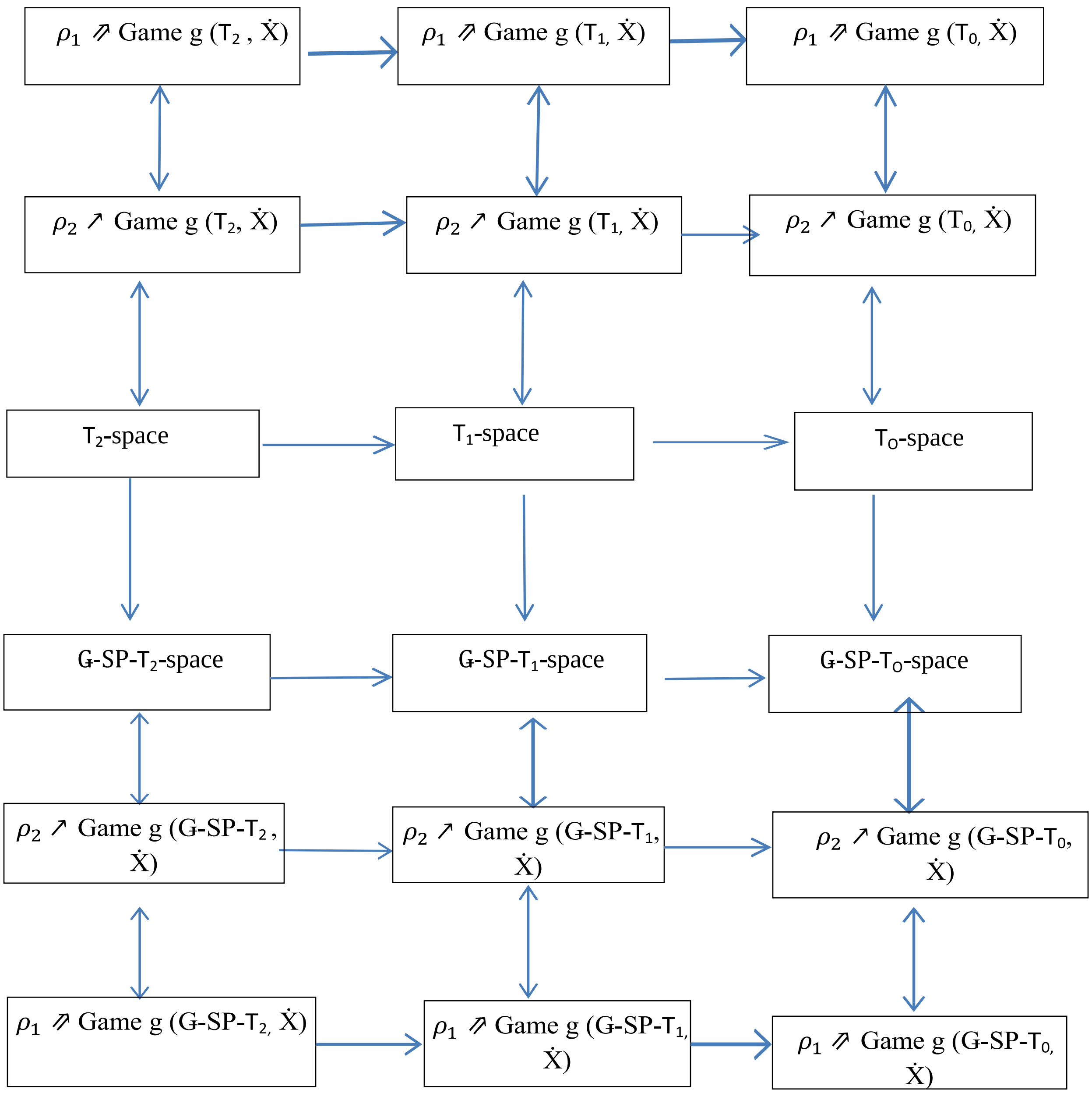
Remark 3. 26: For any space $(\dot{X}, \mathcal{T}, G)$:

i. If $\rho_2 \nearrow$ a Game g (G-SP-$\mathcal{T}_{i+1}$, $\dot{X}$) then $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_i$, $\dot{X}$). whenever i= {0, 1}.

ii. If $\rho_2 \nearrow$ a Game g ($\mathcal{T}_i$, $\dot{X}$) then $\rho_2 \nearrow$ in a Game g (G-SP-$\mathcal{T}_i$,$\dot{X}$). whenever i= {0,1,2}.

The following Diagram 2. 1 clarifies the relationships given in the Remark 3. 26.

Diagram (2. 1)



The winning and losing strategy for any player in Game g (G-SP- $\mathcal{T}_i$, Ẋ) and Game g ($\mathcal{T}_i$, Ẋ).

Remark 3. 27: For any space (Ẋ, $\mathcal{T}$, G):

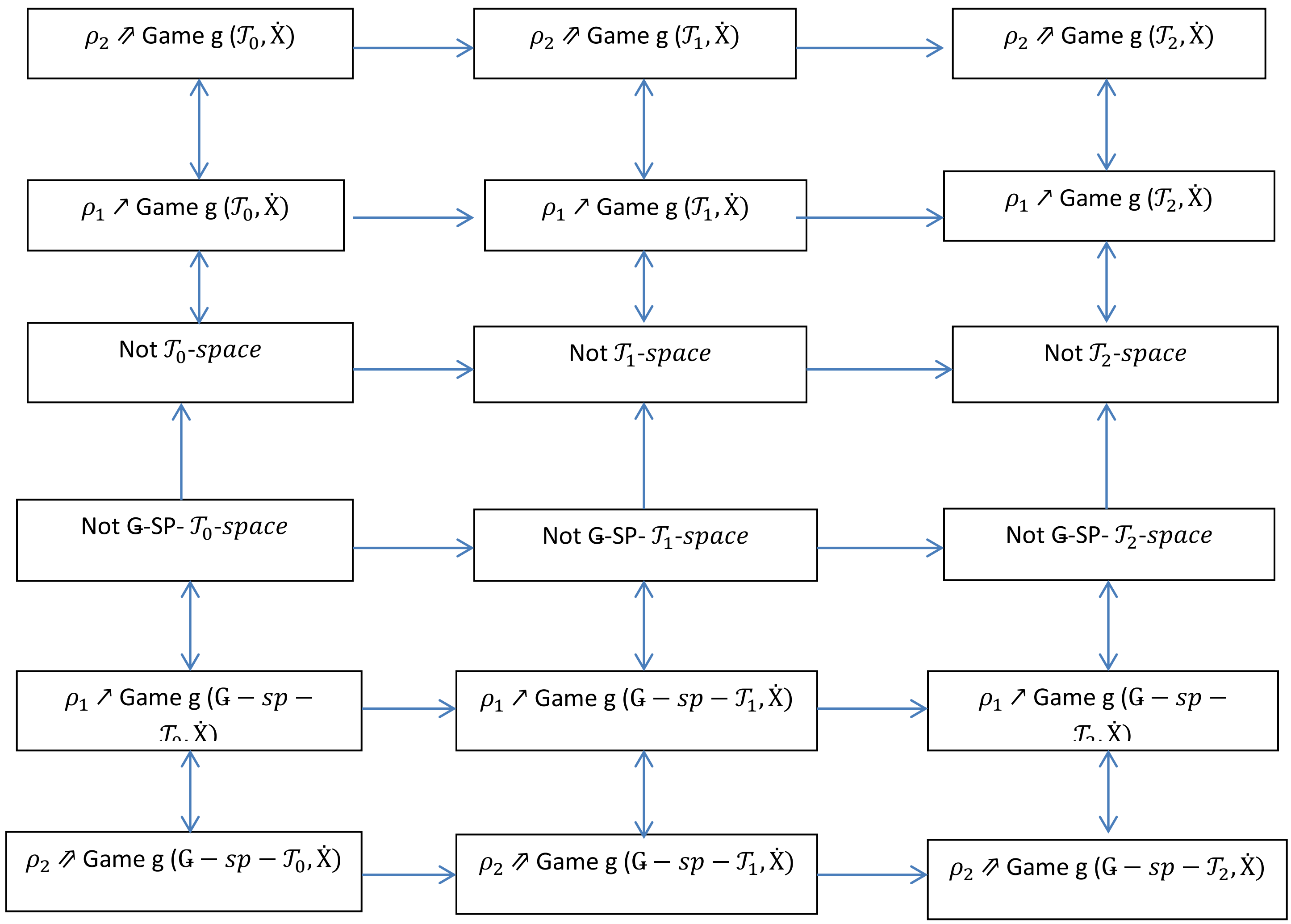
i. If ρ_1 ↗ Game g (G-SP-$\mathcal{T}_{i+1}$, Ẋ), then ρ_1 ↗ Game g ($\mathcal{T}_i$, Ẋ), whenever i= {0, 1}.

ii. If ρ_2 ⇗ Game g (G-SP-$\mathcal{T}_{i+1}$, Ẋ), then ρ_2 ⇗ Game g ($\mathcal{T}_i$, Ẋ), whenever i= {0, 1}.

iii. If $\rho_1 \nearrow$ Game g $(\mathcal{T}_i, \dot{X})$, then $\rho_1 \nearrow$ Game g (G-SP-$\mathcal{T}_i$, $\dot{X}$), whenever i= {0, 1, 2}.

The following Diagram 2. 2 clarifies the relationships given in the Remark 3.27.

Diagram (2. 2)



The winning and losing strategy when $\dot{X}$ is not G-SP-T_i-space and not T_i-space.

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