



Numerical Modeling of Right Circular Cylindrical Fin of Finite Length Subjected to Conduction and Convection Losses in Transient State

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Numerical modeling of right circular cylindrical fin of finite length subjected to conduction and convection losses in transient state

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Abstract: Fins are employed for the purpose of enhancing heat transfer from the base of the component, a known fact. In this paper, the performance calculation for uniform cross-sectional circular fin was done by transient analysis. The necessary numerical equations were derived by applying energy balance. Obtained results were compared with standard results. Variation of temperature over the surface of the fin is assumed to be in one dimension only. The equations were checked for forced convection. Stability criteria were obtained for different boundary conditions. Steady state analysis eliminates the influence of temperature increase when subjected to constant heat flow. This condition was considered, and calculations were done for nodes as well as the complete system. The performance parameters for extended surfaces such as effectiveness and efficiency also vary from steady state to transient state. In this context the energy balance for fin was made at macro and micro levels i.e., by finite difference method employing nodes and overall energy balance.

Keywords: Fin of prescribed length, numerical modeling, transient analysis, forced convection, steady state analysis, simulation, convective heat transfer coefficient, effectiveness and efficiency, energy balance.

Nomenclature

r	– Sheetradius of the fin (m)
Δx	– Nodal distance (m)
ρ	– Density of material (kg/m^3)
c_p	– Specific heat of material (kJ/kg-K)
Δt	– Time (s)
T_1^p	– First nodal temperature at present instant ($^{\circ}\text{C}$)
T_2^p	– Second nodal temperature at present instant ($^{\circ}\text{C}$)
T_3^p	– Third nodal temperature at present instant ($^{\circ}\text{C}$)
T_4^p	– Fourth nodal temperature at present instant ($^{\circ}\text{C}$)

- T_5^p – Fifth nodal temperature at present instant ($^{\circ}\text{C}$)
 T_6^p – Sixth nodal temperature at present instant ($^{\circ}\text{C}$)
 T_1^{p+1} – First nodal temperature at next instant ($^{\circ}\text{C}$)
 T_2^{p+1} – First nodal temperature at next instant ($^{\circ}\text{C}$)
 T_3^{p+1} – First nodal temperature at next instant ($^{\circ}\text{C}$)
 T_4^{p+1} – First nodal temperature at next t instant ($^{\circ}\text{C}$)
 T_5^{p+1} – First nodal temperature at next instant ($^{\circ}\text{C}$)
 T_6^{p+1} – First nodal temperature at next instant ($^{\circ}\text{C}$)
 k – Thermal conductivity of the material (W/m-K)
 h – Convective heat transfer coefficient (W/m²-K)
 T_{∞}^p – Surrounding atmosphere temperature ($^{\circ}\text{C}$)
 F_0 – Dimensionless time, Fourier number
 l – Length of the fin (m)
 η – efficiency of the fin
 ε – effectiveness of the fin
 f – shape factor of the fin corresponding to duct
 ϵ – Emissivity of the fin.
 $p1 = \frac{2h}{r\rho c}$, constant
 $p2 = \frac{k}{l\rho c}$, constant.
 T_{avg} – Average surface temperature ($^{\circ}\text{C}$)

1. Introduction

It is familiar that the purpose of fins is to transfer the heat from the respective heated component to the surrounding atmosphere. The applications of fins are extended to thermal as well as to electrical applications such as transformers in substations, circuit boards, heat sinks, heat exchangers, hydrogen fuel cells, etc. All these above applications are mostly influenced by the convective heat transfer coefficient and effectiveness of the fin. But these parameters alter transiently. During calculation of efficiency and effectiveness of the fins, the convective heat transfer coefficient is assumed to be constant throughout the surface, but the known fact is that it alters along the surface as well as with time (temporal variation). As the base is subjected to constant heat flux but the temperature (temp.) on the surface varies with time longitudinally, after a very long time it achieves steady state.[1] In practical situations there will be not be enough time for complete heat transfer. Hence the only parameter which can effectively transfer heat is the heat transfer coefficient that has to be calculated taking into consideration both the modes viz. conduction and convection.

Most of the heat transfer processes are transient and convective heat transfer coefficient also varies throughout. Hence a steady state process can't be assumed for realistic processes in enhanced surfaces. The performance parameters for extended surfaces such as effectiveness and efficiency also vary from steady state to transient state. In this context the energy balance for fin was made at macro and micro levels i.e., by finite difference method employing nodes and overall energy balance. When compared to steady state, transient state processes give less value for performance parameters as the losses are also considered. [2] In some cases, if the extended surfaces are subjected high temperature bodies then the properties of the material of the fin also alter with time and length. At high temperatures, the fin base material may get high porosity and there may be mass diffusion into the fin material if the surrounding atmosphere is dusty and smoky. For example, at chemical industries such Diammonium Phosphate (DAP) plants the surrounding plant atmosphere is fully heavy mist. In order to provide effective heat transfer from the granulator and dryers to surrounding atmosphere they will be equipped with circumferential fins. These extended surfaces are subjected to continuous thermal and dynamic loading due to which the heat transfer capacity may be affected. Due to continuous thermal

fatigueloading, the material properties may also be altered. For fins the boundary conditions are dependent on geometry of the fins.

In heat exchangers, the fins are subjected to continuous thermal cyclic loads. In such applications, the heat from hot fluid need to be transferred to cold fluid in short time and effectively without losses. That can be achieved by implementing necessary fin suitable to the geometry of the heat exchanger. [3] The fins are classified according to shape as pin fins, rectangular fins, triangular fins, right circular cylinder fins, elliptical fins etc. In steady state analysis, the heat transfer coefficient is assumed to be constant since the variation of properties of surrounding atmosphere, such as Prandtl number, is neglected. Hence the value of convective heat transfer coefficient is calculated by using steady state correlations of fins for various geometries. But in practical scenarios, these parameters alter with temperature, time and length or geometry of the fin. Steady state correlations give accurate values for regular and uniform geometries of the fin. In transient analysis, convective heat transfer coefficient can be computed for any fin of irregular geometry. As the energy balance is applied at each instant of the experimentation, the results obtained will be accurate. It is known that in fins, the heat transfer is the combined phenomenon of convective, conductive and radiation heat transfer. This heat transfer can be enhanced by increasing the surface area of the object or temperature gradient or the convective heat transfer coefficient, and by variation of parameters viz. velocity, viscosity etc. of surrounding atmosphere of the fin. But in most of the scenarios, the temperature gradient and convective heat transfer coefficient are dependent on surrounding atmospheric flow characteristics, ambient temperature, thermal properties of atmosphere and buoyancy effects of the atmosphere. [9]

The effective method to enhance the heat dissipation from the heated object is to increase its surface area. The geometry of the extended surface play vital role in heat transfer. That is if we consider pin fins, the length of the fins over the heated object is completely dependent on the base temperature of the fin. If very long fins are used for heat transfer, instead of dissipation there may be heat accumulation. [5] It means that fins would be acting as insulators. If the length of the fin is short, the necessary amount of heat exchange may not take place and heating the base of the fin beyond the limiting temperature of the fin material may lead to its melting. Such a length of the fin where neither accumulation nor melting takes place is called as effective fin length. [4] For construction of fin, the material should possess high thermal conductivity, low specific heat and high melting point. Presently for heat dissipation from the heated objects, high conductive liquids which change their phase during the heating process are being used. These are referred to as heat sinks. These are advantageous because there is no need of extra material for construction of fins and area or volume occupied by the object is reduced, hence making the design highly compact and efficient. But the main disadvantage is that due to phase transformation, these liquids accumulate high pressure in the pipelines or ducts. These liquids require complete surface contact area with the heated object. [6]

1.1. Assumptions

- The material of the fin is isotropic in nature.
- Temperature variation is along the surface of the cylindrical fin only.
- The radial variation of temperature for the fin is neglected.
- The cross-sectional area of the fin is uniform throughout the length of the fin
- The variation of temperature is in one dimension only, since the cross-sectional area is very small when compared to lateral surface area of the fin.
- The initial temperature of the base of the fin is greater than the surrounding atmospheric temperature.
- Initially all the nodes are at same temperature as that of the atmospheric temperature.
- Heat transfer is taking place in one dimension only.
- Transient analysis is in one dimension.

2. Numerical Modeling

Below picture shows the CAD drawing of right circular cylinder fin with respective dimensions.

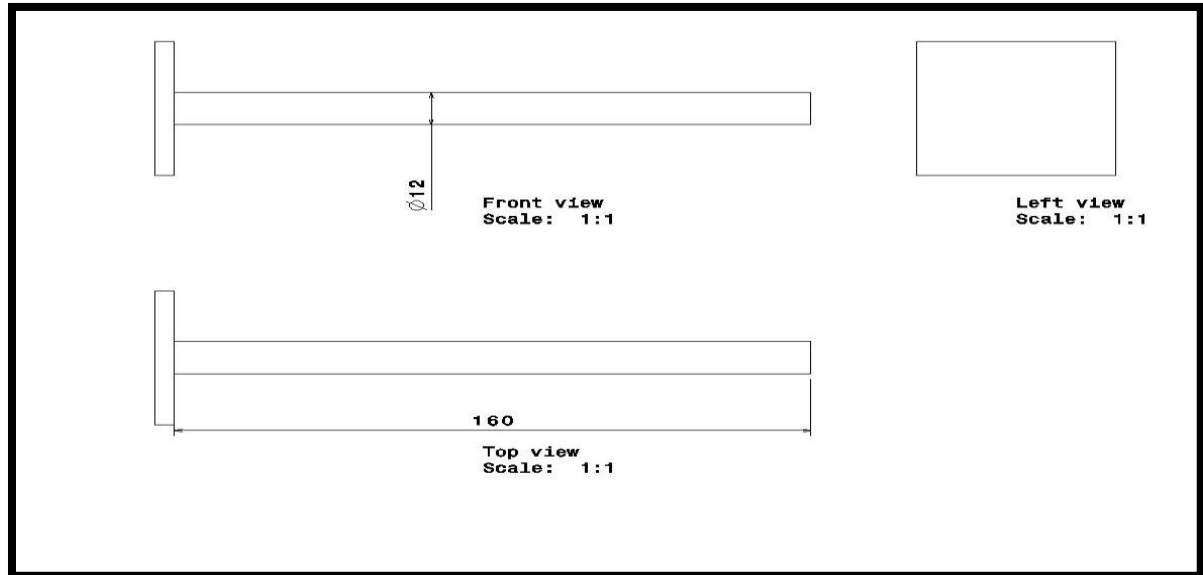


Figure 1. CAD drawing of right circular cylinder fin

2.1. Energy balance

For the shown geometry of the fin, energy balance was written as follows. The input energy was constant heat flux which was applied to the base of the fin. This heat flux was transferred to the body of the fin through conduction. Since the peripheral area of the fin was not insulated, it was free to convect heat from the fin body to surrounding atmosphere. [9] In practice along with conduction and convection, radiation also participates in heat transfer but here the energy balance was written for conduction and convection only. Energy balance for the given right cylindrical fin was written both in microscopic and macroscopic views. Energy balance was written in one dimension as the heat transfer through the fin was assumed to be in one dimension only. [10] The total length of the fin was divided into five equal parts comprising of six nodes. Energy balance for all six nodes was written by their respective boundary conditions using finite difference methods.

Energy at any instant = energy conducted out + energy convected out.

This energy balance equation is valid for the inner nodes. For interior nodes, the heat transfer takes place in axial direction and radial direction only. All the interior nodes had same boundary condition. [8]

$$\frac{\pi r^2 \Delta x \rho c_p}{\Delta t} (T_2^{p+1} - T_2^p) = \frac{k \pi r^2 (T_2^p - T_1^p)}{\Delta x} + h 2 \pi r \Delta x (T_\infty^p - T_2^p)$$

Upon rearranging and non-dimensionalization of the terms we get following equation.

$$T_2^{p+1} = F_0 \left(T_1^p + \frac{2h \Delta x^2 T_\infty^p}{rk} \right) + T_2^p \left(1 + F_0 - \frac{2h \Delta x^2 F_0}{rk} \right)$$

In the above equation, the nodal temperature is a function of non-dimensional parameter Fourier number, convective heat transfer coefficient, surrounding atmospheric temperature and thermal conductivity of the material. For any fin, the heat transfer would be constant after certain time elapse after getting heated. [7] This time can be evaluated by taking previous nodal temperature coefficient greater than or equal to zero.

Stability criteria

$$\left(1 + F_0 - \frac{2h\Delta x^2 F_0}{rk}\right) \geq 0$$

$$F_0 = \frac{1}{\left(\frac{2h\Delta x^2}{rk} - 1\right)}$$

For the nodes at tip of the fin, the boundary condition was all the heat conducted to the tip of the fin was convected out to the atmosphere. This was because at the tip there was no insulation and it was open to atmosphere. Hence the only mode to transfer heat to the surrounding was convection. As mentioned earlier radiation will also take place, but in this analysis only conduction and convection were considered.

Energy at any instant = energy conducted out = energy convected out.

$$\frac{\pi r^2 \Delta x \rho c_p}{\Delta t} (T_6^{p+1} - T_6^p) = h 2\pi r \Delta x (T_\infty^p - T_6^p)$$

By rearranging we get

$$T_6^{p+1} = F_0 T_\infty^p + T_6^p (1 - F_0)$$

Stability criteria for node at the tip

$$(1 - F_0) \geq 0$$

$$F_0 = 1$$

At the base of the fin the heat supplied was equal to heat at that instant.

$$Q = \frac{\pi r^2 \Delta x \rho c_p}{\Delta t} (T_1^{p+1} - T_1^p)$$

Energy balance in macroscopic view was given by equating heat conducted and convected through the fin body to energy available in the fin at any instant.

Complete energy balance equation for the fin

$$\frac{\pi r^2 \Delta x \rho c_p}{\Delta t} \frac{d\theta}{dt} = h 2\pi r l (T_s^p - T_\infty^p) + k \pi r^2 \frac{d\theta}{dx}$$

$$\theta = T_s^p - T_\infty^p$$

T_s^p - Average surface temperature of the fin.

This reduces to

$$\frac{d\theta}{dt} = p_1 \theta + p_2 \frac{d\theta}{dx}$$

This is a single order differential equation which can be solved by variable separable method.

$$p_1 = \frac{2h}{r\rho c}, p_2 = \frac{k}{l\rho c}$$

Upon solving the above equation the resulting equation will be as follows

$$\theta = 8e^{1.024ht*10^{-4}}$$

Reason for non-dimensionalizing the energy balance equation was that the limits can be constrained from 0 to 1, so that numerical modeling can easily be done. The constants in the expression will be deduced into some non-dimensional parameters.

Numerical modeling can be done at micro level and macro level. Micro level numerical modeling helps in knowing the nodal temperatures throughout the length of the fin. If analysis was carried out in two-dimensions, variation of temperatures can be known in radial and axial direction. But the variation of convective heat transfer coefficient must be known at each node. Since cylindrical fin was considered for analysis, two dimensional correlations need to be obtained in polar coordinates. Considering this complication, the temperature in radial direction was assumed to be constant with respect to nodal temperature and surrounding atmosphere temperature.[10]

3. Experimental setup

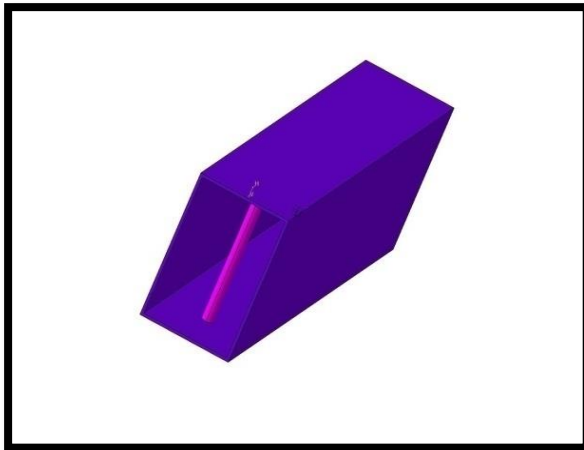


Figure 2. Experimental setup of fin in forced convection

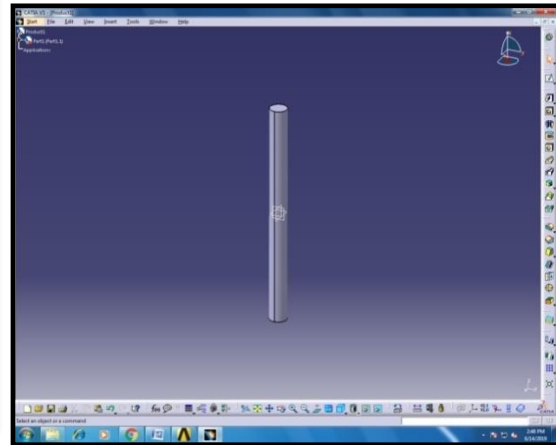


Figure 3. CATIA drawing of fin

The experimental set up consists of a rectangular duct of dimensions 150 x 160 mm extending over a length of 350mm. A brass cylindrical fin of diameter 12mm and length equal to width of the duct was placed at 100mm from one end. To one end of the fin, a constant heat flux was given with help of a heater element, whose heat input was varied by varying the current and voltage to the equipment. By the above formulations and stability criteria, the time gap between two consecutive readings was calculated as 5minutes i.e., for every five-minute readings of temperature at all the six nodes taken by distributor switch were taken at constant heat input. The same procedure was carried out for different heat inputs. By employing the nodal temperatures in the respective equations, convective heat transfer coefficient, and the values of effectiveness and efficiency of the fin were calculated. The flow of air from the blower over the cylindrical fin was cross flow. The blower was operated at a pressure of 5mm of Hg.

Thermal conductivity $k = 110.7 \text{ W/m-K}$
 Diameter of the fin = 12mm
 Density of the fin material brass = 8565 kg/m^3
 Specific heat of fin material = 380 J/kg-K
 Length of the fin = 150mm

Equipment used for experimentation was aged, hence the values of effectiveness and efficiency were not appreciable but numerical modeling suggested in this paper had satisfied the criteria.

4. Results and Discussions

Complete experiment was carried out at two different heat input values 17W and 20W. Plots for performance parameters along with convective heat transfer coefficient and average surface temperature were done and they were compared with steady state values.

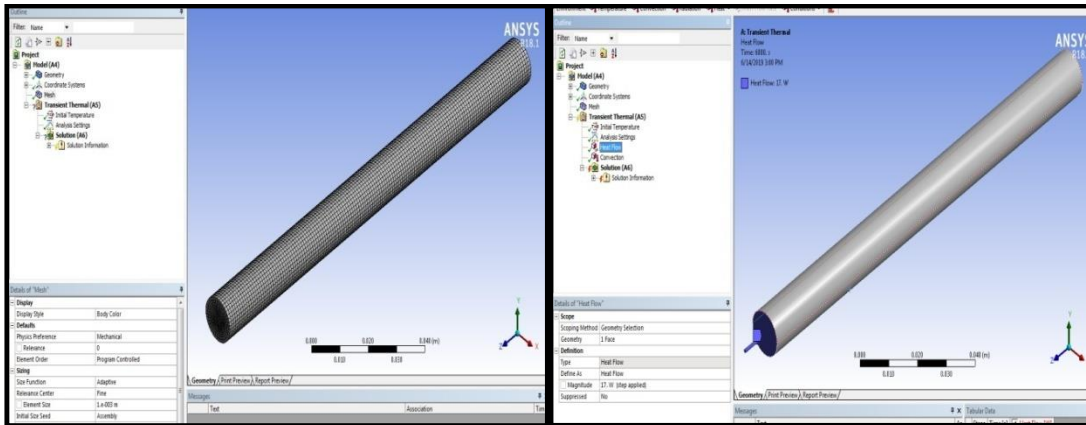


Figure 4. Meshing of fin in ANSYS 18.1

Figure 5. Applying heat load of 17W to fin base

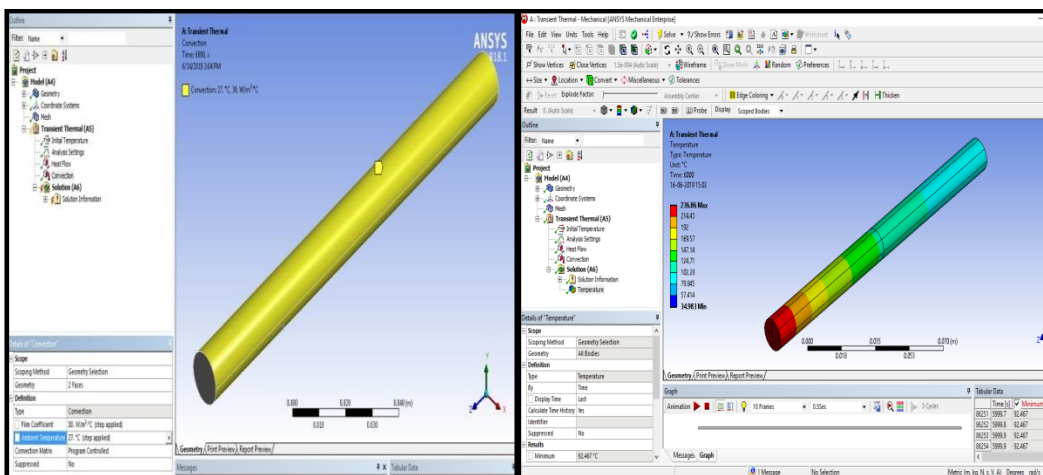


Figure 6. Application of h 30W/m²-K

Figure 7. Temperature variation for 6000sec

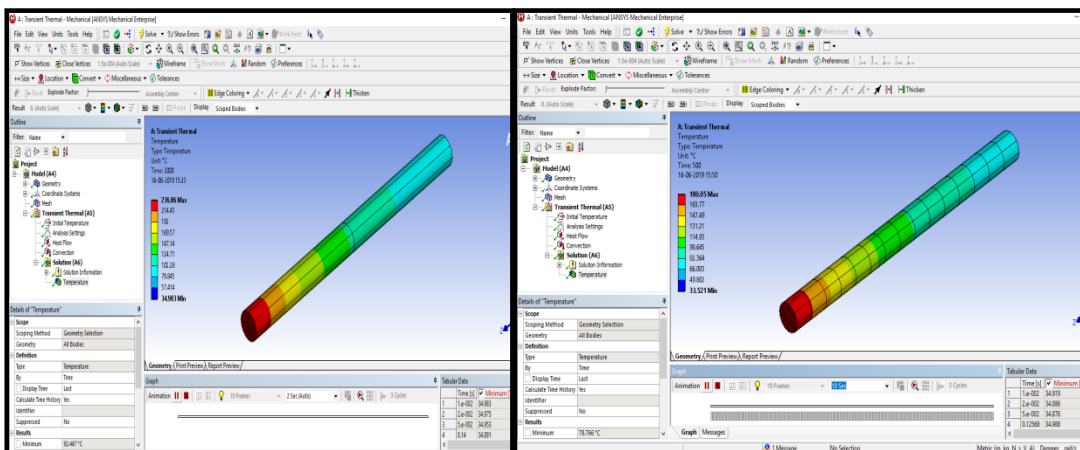


Figure 8. Temp. variation for 2000sec 17W

Figure 9. Temp. variation for 2000sec with 20 W

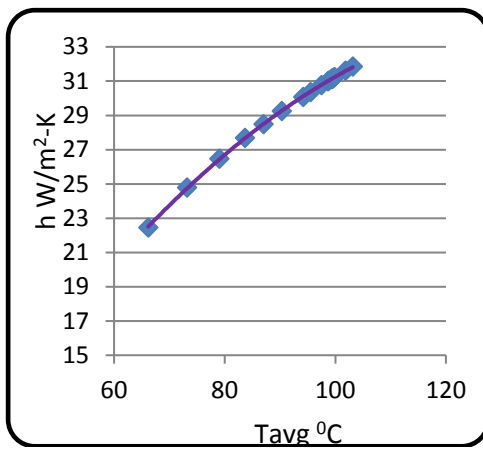


Figure 10. Variation of h with T_{avg}

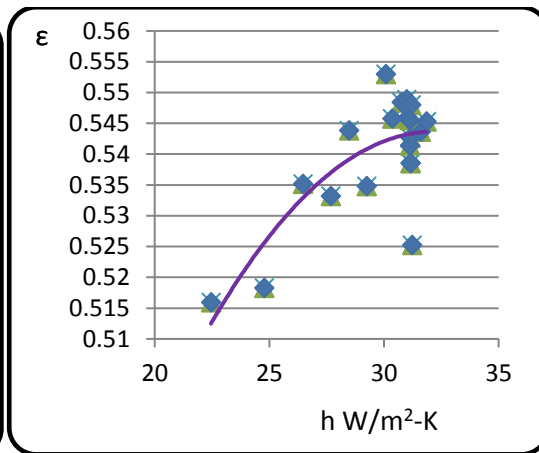


Figure 11. Variation of h with ϵ

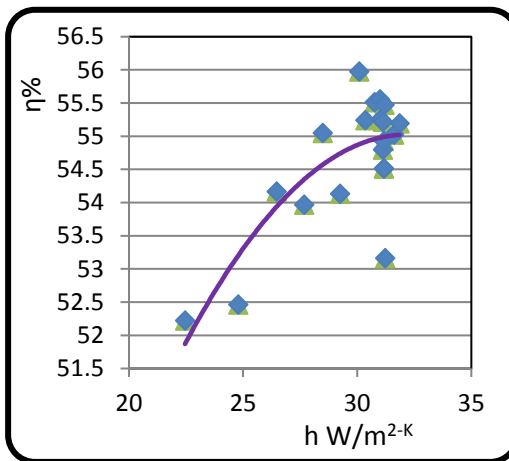


Figure 12. Variation of h with η 17W

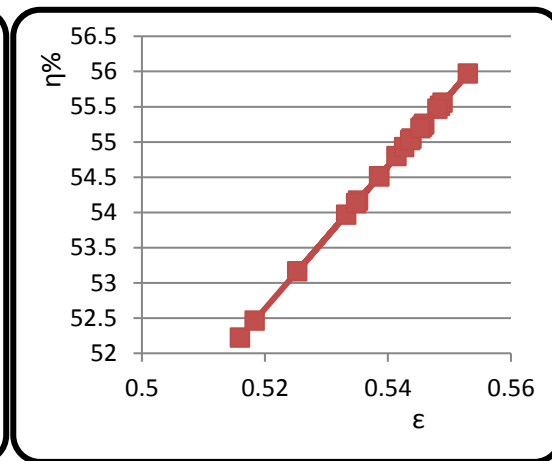


Figure 13. Variation of ϵ with η 17W

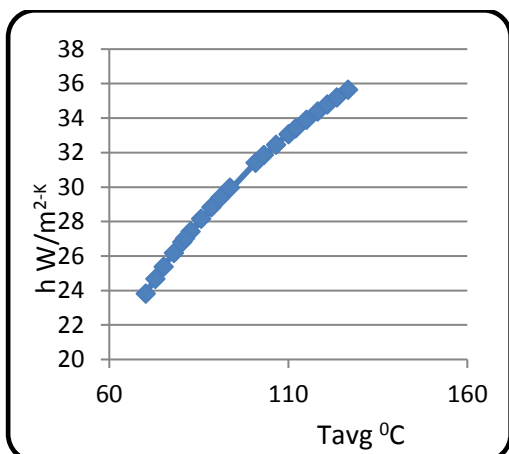


Figure 14. Variation of h with T_{avg} at 20W

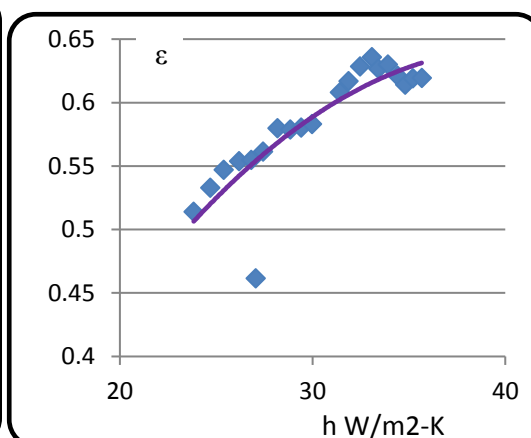


Figure 15. Variation of ϵ with h at 20W

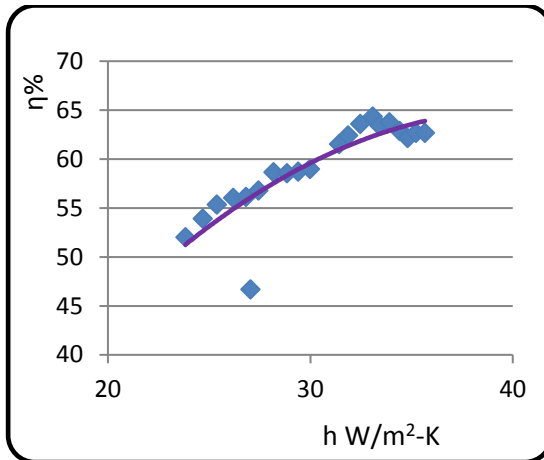


Figure 16. Variation of h with η 20w

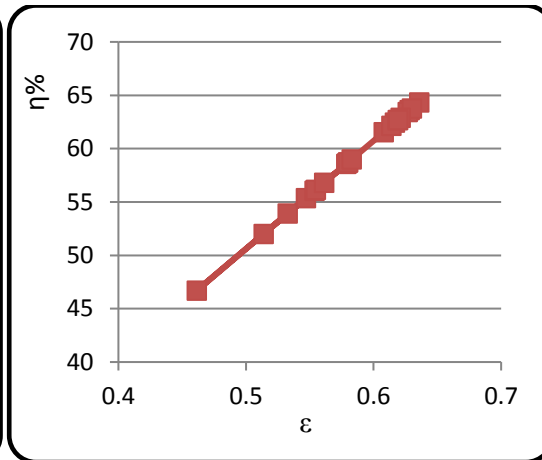


Figure 17. Variation of ϵ with η 20W

Above graphs depicts the transient behavior of fin for various performance parameters. As the heat input to fin goes on increasing, the convective heat transfer coefficient, effectiveness and efficiency were also increasing but the maximum temperature was limited depending upon the material and cross-sectional area of the fin. It was observed that the change in convective heat transfer coefficient and average surface temperature was less, however the values for effectiveness and efficiency had changed appreciably on increase of heat input. Efficiency of the fin increased as heat input increased, and minimum value of efficiency also increased with increase of heat flux at base of the fin. CATIA was used for drawings and ANSYS for necessary simulations. The procedure was repeated for two different thermal loads for constant flux at the base of the fin. It was observed that as the time of constant heat flux increased, the convective heat transfer coefficient also increased. The boundary conditions taken in the simulation were lateral surface area and cross-sectional area at the tip of the fin was subjected to convection only. The influence of radiation heat transfer was neglected in this analysis. The mesh size adopted for both analyses was 0.01. Complete numerical simulation was done at constant ambient temperature. Coarse size followed in simulation in of fine type. The variation of effectiveness with efficiency of the fin for both heat inputs was almost linear i.e., as the efficiency of the fin increased effectiveness also increased. The graph between convective heat transfer coefficient and efficiency of the fin follows a quadratic variation. The variation of effectiveness and convective heat transfer coefficient also obeys the quadratic polynomial fashion. The order of variation of average surface temperature of the fin to convective heat transfer coefficient was almost linear but the exact fitting of the curve was a polynomial equation of order 2.

5. Conclusions

- As the base temperature of the fin increased, the average convective heat transfer coefficient also increased.
- As the heat input to base of the fin increased, effectiveness and efficiency of the fin also increased.
- Performance properties values were less in transient condition when compared to those in steady state.
- The convective heat transfer coefficient was completely dependent on the surface temperature of the fin and velocity of air from the blower.
- Heat transfer can be enhanced even by changing the flow rate of blower, but power consumption increased as the discharge of the blower increased.

Table 1. Average value of the performance parameters of the fin at different heat inputs

Property	Heat input 17W	Heat input 20W
Average surface temperature(⁰ C)	93.2833	96.944
Average convective heat transfer coefficient (W/m ² -K)	29.698	30.187
Average value of m	13.579	13.618
Average value of effectiveness	0.5396	0.584
Average value of efficiency (%)	54.617	59.114

The above table describes data about the average values of all performance influencing parameters of fin such as surface temperature, heat transfer coefficient, fin parameter, efficiency and effectiveness. It was observed that as heat input i.e., heat flux at the base of the fin increased, values of all the parameters increased appreciably. The rate of heat transfer through the fin by conduction and convective modes increased as the base temperature increased. But the base temperature had a constraint of maximum value corresponding to fin material melting temperature.

6. Future scope

Since every material above 0⁰K radiates heat, the phenomenon of radiation can also be considered in order to get more accurate results. As the temperature increases, the radiation intensity also increases. The necessary energy balance equation at macro and micro level is as follows.

Energy at any instant = energy conducted out + energy convected out + energy lost due to radiation

This energy balance equation is valid for the inner nodes.

$$\frac{\pi r^2 \Delta x \rho c_p}{\Delta t} (T_2^{p+1} - T_2^p) = \frac{k \pi r^2 (T_1^p - T_2^p)}{\Delta x} + h 2 \pi r \Delta x (T_\infty^p - T_2^p) + \sigma \in 2 \pi r \Delta x (T_1^{p4} - T_2^{p4})$$

But for the nodes at tip of the fin the boundary condition will be as follows

Energy at any instant = energy conducted out = energy convected out.

$$\frac{\pi r^2 \Delta x \rho c_p}{\Delta t} (T_6^{p+1} - T_6^p) = h 2 \pi r \Delta x (T_\infty^p - T_6^p) = \sigma \in 2 \pi r \Delta x (T_6^{p4} - T_\infty^{p4})$$

Complete energy balance

$$\frac{\pi r^2 \Delta x \rho c_p}{\Delta t} \frac{d\theta}{dt} = h 2 \pi r l (T_s^p - T_\infty^p) + k \pi r^2 \frac{d\theta}{dx} + \sigma \in 2 \pi r \Delta x (T_s^{p4} - T_\infty^{p4})$$

When radiation is also considered, some of the parameters influences the rate of heat transfer such as Stefan Boltzmann constant, shape factor of the geometry and emissivity of the material of the fin. Since the fin is completely enclosed in the duct, the shape factor of fin corresponding to duct is taken as unity. If the emissivity of the material is taken from the standard values, the heat transfer coefficient will be combined parameter of convective and radiative heat transfer coefficient, which is called as overall heat transfer coefficient.

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