Interaction Between the VaR and the Interest Rate of Cash Flow

Mostafa El Hachloufi, Driss Ezouine and Mohammed El Haddad

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[1]EL HACHLOUFI Mostafa
Faculty of Juridical, Economical and Social Sciences, University of Mohamed V, Rabat, Morocco
elhachloufi@yahoo.fr

EZOUINE Driss
Faculty of Juridical, Economical and Social Sciences, University of Mohamed V, Rabat, Morocco
ezouinedriss@gmail.com

EL HADDAD Mohammed
Faculty of Juridical, Economical and Social Sciences, University of Mohamed V, Rabat, Morocco
mo.haddad@hotmail.com

ABSTRACT

In this paper, we propose an approach to study the impact of the interest rate on the risk of variation in cash flows measured by the value at risk (VaR) using stochastic processes and ALM technics.

This approach provides a decision-making tool for manage asset, liability funds to bankers insurers and all companies operating in the financial sector.

CCS CONCEPTS

Financial Mathematics and Statistics.

Keywords

Interest rates; VaR; cash flow; ALM Technics; Stochastic Processes.

1. INTRODUCTION

ALM is one of the main tools used to help solve rate variation problems in financial institutions such as banks and insurance companies. He plays a very important role in managing the various activities of the financial institute.

Appropriate liquidity and balance sheet management are a key factor in ensuring the activity of financial institutions and are a tool for managers to make decisions about risk management with variations in the interest rate.

The activities of companies, whether banks, insurance companies or for-profit corporations, generate cash flows affecting the balance sheet as assets or liabilities. This financial flow and its risk are influenced by the variation in the interest rate. In the case of a positive or negative variation, it will have an impact on the assets, liabilities or both at the same time.

The objective of this paper is to study this influence by treating the impact of the change in the interest rate on the risk of this flow

2. INTEREST RATE

The interest rate is defined as the economic remuneration of time. This is the amount a borrower is willing to pay to his lender in addition to the capital, and it is based on the credit risk of that borrower.

The evolution of the interest rate can be modeled by several stochastic processes whose Vasicek process or model is the most popular. This stochastic process is called the process of return to the mean.

The Vasicek model assumes that the current short-term interest rate is known, while the future values of this rate follow the following equation:

$$dr_t = \eta\left(\bar{r} - r_t\right)dt + \sigma dz_t \qquad (1)$$

Where :

- η : is the rate of return of the interest rate to the average.
- $\bar{r}$: is the average interest rate.
- σ : is the volatility of the interest rate which is assumed to be independent of r_t
- z_t : is a Brownian movement such as $dz_t = \varepsilon_t \sqrt{dt}$ avec $\varepsilon_t \sim N\left(0,1\right)$

3. THE VAR OF FINANCIAL FLOW

The VaR is a measure of risk most widely used in financial markets to quantify the maximum loss on a portfolio for a given horizon and confidence level. It depends on three elements:

- The distribution of the portfolio's profits and losses for the holding period.
- The level of confidence.
- The holding period of the asset.

Analytically, the VaR with time horizon t and the threshold probability α the number $VaR(t,\alpha)$ such as:

$$\mathrm{P}\left[\Delta R \leq VaR(t,\alpha)\right] = \alpha \qquad (2)$$

where :

- t : Horizon associated with VaR which is: 1 day or more than one day.
- α : The probability level is typically 95%, 98% or 99%.

The financial flow $F_i, i=1,...,n$ from a company E is a set of n amounts received or paid by it at different times $t_i, i=1,...,n$ to the future. In other words: $\left\{\left(F_i, t_i\right)/\, i=1,...,n\right\}$, where:

- F_i is the amount of the it^h movement of the financial flow.
- t_i is the maturity of the it^h movement of the financial flow.

Whose interest rate corresponds to the t_i is r_{t_i}. Interest rates $r_{t_i}, i=1,...,n$ are assumed to be independent.

The present value of this financial flow is given by:

$$V_r = \sum_{i=1}^{n} F_i r_{t_i} = F_1 r_{t_1} + F_2 r_{t_2} + ... + F_n r_{t_n}$$

Consider two future flows: assets A and liabilities P given by :

$$A = \left\{\left(A_i, r_{t_i}\right),\ \ \mathrm{i}=1,...,n\right\}),\ \mathrm{P} = \left\{\left(P_i, r_{t_i}\right),\ \ \mathrm{i}=1,...,n\right\}$$

The surplus relative to the couple of flow $\{A,P\}$ in relation to the interest rate r_{t_i}, $\mathrm{i}=1,...,n$, noted S_{r_t} , is given by:

$$S_{r_t} = \sum_{i=1}^{n} \left(A_i - P_i\right) \times r_{t_i}$$

Or $S_{r_t} = \sum_{i=1}^{n} AP_i \times r_{t_i} = \sum_{i=1}^{n} F_i \times r_{t_i}$

where $F_i = AP_i = \left(A_i - P_i\right)$

Assuming that the interest rate follows a Vasicek process, i.e :

$$dr_t = \eta\left(\bar{r} - r_t\right)dt + \sigma dz_t$$

Thus, we can develop this model which is used to describe the interest rate as follows:

$$d\left(e^{\eta t} r_t\right) = e^{\eta t} dr_t + r_t \eta e^{\eta t}$$

$$\Rightarrow e^{\eta t} dr_t = d\left(e^{\eta t} r_t\right) - r_t \eta e^{\eta t}$$

In other, $dr_t = \eta\left(\bar{r} - r_t\right)dt + \sigma dz_t$

$$\Rightarrow e^{\eta t} dr_t = e^{\eta t}\eta\left(\bar{r} - r_t\right)dt + e^{\eta t}\sigma dz_t$$

Then we get:

$$d\left(e^{\eta t} r_t\right) = \eta \bar{r} e^{\eta t} dt + e^{\eta t}\sigma dz_t$$

$$\Rightarrow r_t = r_0 e^{-\eta t} + \int_0^t \eta e^{-\eta(t-s)} \bar{r} ds + \sigma \int_0^t e^{-\eta(t-s)} dz_s$$

So the interest rate r_t can be expressed as follows:

$$r_t = \bar{r} + \left(r_0 - \bar{r}\right)e^{-\eta t} + \sigma \int_0^t e^{-\eta(t-s)} dz_s \qquad (3)$$

Knowing that z_t is a process such as $dz_t = \varepsilon_t \sqrt{dt}$ with $\varepsilon_t \rightarrow N\left(0,1\right)$, so dz_t follows the normal distribution and the variable $\left(\sigma \int_0^t \eta e^{-\eta(t-s)} dz_s\right)$ also follows the normal distribution.

The term $\bar{r} + \left(r_0 - \bar{r}\right)e^{-\eta t}$ is not a random term then $r_t | r_0$ is a random variable that follows the normal distribution. Using Iso isometry we find that:

$$E\left[\left(\sigma \int_0^t \eta e^{-\eta(t-s)} dz_s\right)\right] = 0$$

and

$$E\left[\left(\sigma \int_0^t \eta e^{-\eta(t-s)} dz_s\right)^2\right]$$

$$= \int_0^t \left(\sigma e^{-\eta(t-s)}\right) ds = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right)$$

Then the mean and the variance are respectively:

- $E\left(r_t | r_0\right) = \bar{r} + \left(r_0 - \bar{r}\right)e^{-\eta t}$
- $V\left(r_t | r_0\right) = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right)$

So the random variable $r_t | r_0$ follows the normal distribution of mean and variance respectively $\bar{r} + \left(r_0 - \bar{r}\right)e^{-\eta t}$ et $\frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta t}\right)$, i.e :

$$r_t \rightarrow N\left(\bar{r} + \left(r_0 - \bar{r}\right)e^{-\eta t}, \sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta t}\right)}\right)$$

Knowing that:

$$V_{r_t} = \sum_{i=1}^{n} F_i r_{t_i} = F_1 r_{t_1} + F_2 r_{t_2} + ... + F_n r_{t_n}$$

so

$$dV_{r_t} = \sum_{i=1}^{n} F_i dr_{t_i}$$

and

$$dr_t = \eta(\bar{r} - r_t)dt + \sigma dz_t$$

so

$$dV_{r_t} = \sum_{i=1}^{n} F_i dr_{t_i} = \sum_{i=1}^{n} F_i \left[\eta\left(\bar{r} - r_{t_i}\right)dt_i + \sigma dz_{t_i}\right]$$

$$\Rightarrow dV_{r_t} = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)dt_i + F_i \sigma dz_{t_i}$$

$$\Rightarrow \Delta V_{r_t} = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)\Delta t_i + F_i \sigma \sqrt{\Delta t_i}\,\varepsilon_{t_i}$$

So $E\left(\Delta V_{r_t}\right) = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)\Delta t_i$

and

$$V\left(\Delta V_{r_t}\right) = \sum_{i=1}^{n} (\sigma F_i)^2 \Delta t_i$$

Let $\alpha_t = \sum_{i=1}^{n} F_i \eta\left(\bar{r} - r_{t_i}\right)\Delta t_i$

and

$$\beta_t = \sum_{i=1}^{n} (\sigma F_i)^2 \Delta t_i$$

then $\Delta V_{r_t} \to N\left(\alpha_t, \sqrt{\beta_t}\right)$

The calculation of the VaR at α is given by the following equation:

$$P\left(\Delta V_{r_t} \le VaR_\alpha\right) = \alpha$$

Knowing that $\Delta V_{r_t} \to N\left(\alpha_t, \sqrt{\beta_t}\right)$

therefore $P\left(\frac{\Delta V_{r_t} - \alpha_t}{\sqrt{\beta_t}} \le \frac{VaR_\alpha - \alpha_t}{\sqrt{\beta_t}}\right) = \alpha$

$$\Rightarrow \frac{VaR_\alpha - \alpha_t}{\sqrt{\beta_t}} = \tau_\alpha .$$

Thus

$$VaR_\alpha = \sum_{i=1}^{n} \eta F_i \left(\bar{r} - r_{t_i}\right)\Delta t_i + \tau_\alpha \sigma^2 \sum_{i=1}^{n} F_i^2 \Delta t_i \quad (4)$$

4. CONCLUSION

In this paper, we have developed an approach based on a mathematical formula to study the impact of the interest rate on the risk of the financial flow using value at risk as a risk measure and ALM to express the variability of a company's financial flows.

This approach allows us to evaluate the risk of financial flows in relation to the variation in the interest rate which gives rise to a decision-making tool for the management of funds, whether at the level of the assets or the liabilities of the company.

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